Impact of Buses on the Macroscopic Fundamental Diagram of Homogeneous Arterial Corridors

Felipe Castrillon, Jorge Laval

Abstract

This paper proposes a modification of the method of cuts to estimate the effect of bus operations on the Macroscopic Fundamental Diagram of long arterial corridors with uniform signal parameters and block lengths. It is found that the impact of buses can be adequately captured by the theory of moving bottlenecks using only the bus average free-flow speed. This speed considers the effects of signals and stops but not traffic conditions and therefore can be calculated endogenously. A formulation is developed to approximate the bus average free-flow speed as a function of bus operational parameters. We also found that buses produce an additional capacity restriction that is similar to the "short block" on networks without buses and that can severely reduce corridor capacity. The proposed method is validated with numerical approximations of the corresponding kinematic wave problem.

Keywords: bus operations, moving bottleneck, macroscopic fundamental diagram

1. Introduction

The Macroscopic Fundamental Diagram (MFD) can be a valuable tool to monitor and control congestion on urban networks. First proposed by Daganzo (2007), the MFD relates the average flow and density of a network as an invariant property of the system. This defined relationship makes it possible to obtain basic traffic performance measures by simply knowing the number of vehicles in the network. Currently, there are three main approaches to estimate the MFD. The first one is micro-simulation method which was first verified by (Geroliminis and Daganzo, 2008) on a large network in Downtown San Francisco. Also, there is the variational theory (VT)
network approach, which is numerical (Daganzo, 2005a,b). Finally, we have
the analytical approach, or the "method of cuts" (MOC) which has been de-
developed for homogeneous corridors (Daganzo and Geroliminis, 2008) and later
extended to heterogenous corridors (Laval and Castrillon, 2015). Geroliminis
and Boyaci (2012) use the analytical approach to perform sensitivity analysis
of topological features (block lengths, signal parameters) on the MFD.

The approaches above focus on a homogeneous stream of cars but do
not include the effects of other modes of transportation. To address this,
a few research studies have extended the models to incorporate a mixed
stream of car and bus operations. From a micro-simulation perspective,
Geroliminis et al. (2014) demonstrate the existence of a bi-modal MFD of
Downtown San Francisco with real-life bus schedule inputs. Other micro-
simulated approaches of mixed traffic include Castrillon and Laval (2013);
Xie et al. (2013).

Boyaci and Geroliminis (2011) extend VT networks to include bus stops as
temporary bottlenecks blocking one lane. The authors explore a few corridor
configurations and find that bus capacity varies greatly with changing input
parameters. Xie et al. (2013) extend this work to incorporate the moving
bottleneck effect of buses (Gazis and Herman, 1992; Newell, 1993; Munoz
and Daganzo, 2002) by reducing capacity by one lane for a particular block
during the time that the bus is active on that block similar to Daganzo
and Laval (2005). These numerical methods are exact and useful tools to
perform simulations but provide limited insight into how input parameters
(bus operation parameters, signal timings, block length) affect the output
capacity of a system. Eichler and Daganzo (2006) and (Xie et al., 2013)
also focus on the analytical methodology. The authors incorporate buses to
the MOC by introducing a moving bottleneck cut to the MFD, under the
assumption that the average bus operating speed is known.

Other multi-modal approaches focus on passenger flow (Geroliminis et al.,
2014; Chiabaut, 2015a) and/or dedicated bus lanes (Eichler and Daganzo,
2006; Chiabaut et al., 2014). Chiabaut (2015b) investigate the effects of
double-parked trucks, which provide similar but longer lasting effects than
approach to allocate road space between cars and buses using optimization
methods. Gonzales and Daganzo (2012) extends the morning commute prob-
lem to include multi-modal operations. Zheng et al. (2013) investigates a 3D-
MFD for bi-modal urban traffic using a commercial micro-simulation model.
Gayah et al. (2016) analyses the impact of obstructions such as bus stops on
the capacity of nearby signalised intersections using variational theory.

A common feature of the above references seeking analytical insight is the assumption that the bus operating speed is known exogenously. In this paper we propose a method to overcome this limitation. It turns out that a good approximation of the impact of buses is based on the bus average free-flow speed. This speed considers the effects of signals and stops but not traffic conditions and therefore can be calculated endogenously. A formulation is developed to approximate the bus average free-flow speed as a function of bus operational parameters. This realization allowed us to propose a modification of the method of cuts (mMOC), which is simple, parsimonious and uses only four parameters. Compared to the analytical component in (Xie et al., 2013), our method (i) approximates the average bus operating speed endogenously using renewal theory, and (ii) captures what we call the “bus short-block effect”, which is a result of the interactions between buses and traffic signals.

The remainder of this paper is organized as follows. Following a brief background in section 2, the problem is defined in section 3. Section 4 uses simulation sensitivity analysis to understand the effect of key parameters on the MFD and to provide key insight to formulate the proposed mMOC method in section 5. Section 6 gives a discussion and outlook.

2. Background

In this section we will review the basic building blocks for the proposed methods: the theory of moving bottlenecks and the method of cuts.

Hereafter, a lane will be assumed to follow a triangular fundamental diagram with capacity $Q$, jam density $\kappa$, free-flow speed of cars $u$ and congestion wave speed $-w$, and critical density $k_c$; notice that only three parameters are required while the rest can be derived.

2.1. Moving bottlenecks

The moving bottleneck model was first reported in Gazis and Herman (1992), but it was Newell (1998) who puts it in the context of kinematic wave theory. The theory is particularly useful when the fundamental diagram on the roadway is assumed to be triangular in shape. The empirical validation can be found in Munoz and Daganzo (2002), while numerical methods for incorporation into simulation models are presented in Daganzo and Laval (2005); Laval and Daganzo (2006).
A moving bottleneck creates a free-flow traffic state downstream of its trajectory denoted by $D$ and a congested traffic state upstream given by $U$. These traffic states are plotted on the fundamental diagram in Fig. 1(a).

The downstream flow can be assumed to be the capacity of the remaining unblocked lanes $Q_D = Q(n - 1)$ where $n$ is the number of lanes. A bus traveling at speed $v$ is represented by the line between traffic states $D$ and $U$ in the figure, which acts as an upper bound to the fundamental diagram as vehicles cannot pass the bus on one of the lanes. An example of the moving bottleneck effect on the time-space diagram is shown in Fig. 1(b), where a traffic state $A$ is present before and after the bus enters the segment at location $x_0$.

2.2. Method of Cuts

According to VT, the solution to a kinematic wave problem can be obtained by solving:

$$N(P) = \inf_{B \in \beta_P} \{N(B) + \Delta_{BP}\},$$

(1)

where $N(P)$ is the cumulative count of vehicles at location $x$ by time $t$, $P = (t, x)$ is a generic point, $\beta_P$ is the boundary data in the domain of dependence of $P$, $\Delta_{BP}$ is the maximum number of vehicles that can cross the minimum path connecting boundary point $B = (t_B, x_B)$ and point $P$.

Daganzo and Geroliminis (2008) express (1) in terms of the steady state flow, i.e. $q = N(P)/t, t \to \infty$, which gives the method of cuts:

$$q = \min_{s} \{sk + R(s)\},$$

(2)
Figure 2: (a) Initial value variational theory problem with constant density on the boundary. For each boundary point $B$, the minimum path is obtained from all valid paths. (b) Each minimum path corresponds to a upper boundary "cut" to the MFD

where $k$ is the initial density, $s$ is the average speed of a valid path and $R(s) = \Delta_{BP}/t$ is the maximum passing rate of the minimum path. Therefore, the MFD is the lower envelope of the family of "cuts" $q_s(k) = sk + R(s)$, as shown on Fig. 2b.

3. Problem definition

Consider an arterial corridor consisting of a large sequence of traffic signals with common green time $g$, red time $r$, cycle time $c = r + g$, and offset between two neighboring signals, $o$. The distance between two traffic signals, or simply block lengths, is $l$, also constant. Buses travel the entire corridor with inter-arrival times obtained from a Poisson process with mean headway $h$.

Buses travel at a maximum speed $v \leq u$ when unobstructed by downstream traffic, red lights, or bus stops. These stops are located at each block and the bus will stop in each one with a probability $p$, for a (deterministic) dwell time $d$.

A key variable will be the bus average free-flow speed, $\bar{v}$, where "free-flow" means that the bus is constrained only by red lights and bus stops, not by downstream traffic.
The default parameter values are given in Table 1; they are used throughout the paper unless otherwise stated.

### 4. Simulation experiments

In this section we run three simulation experiments to see how the different parameters affect the shape of the MFD. The simulation tool is described in Appendix A, and gives the exact numerical solution of the problem in the previous section.

Each flow-density data point obtained in this section corresponds to an individual simulation, and the flow-density measure is taken after reaching steady-state. Different traffic states in the free-flow branch are obtained by running the simulation with initial and boundary conditions corresponding to a density ranging from zero to the critical density. To obtain the congested branch the initial and boundary conditions are maintained at capacity while limiting the exit rate of the last intersection on the corridor. In this case the entire corridor will be homogeneously congested in the steady-state, as required by the theory, because the queue generated at the downstream exit

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Default value</th>
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</thead>
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<tr>
<td>cell size</td>
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</tr>
<tr>
<td>time step</td>
<td>( \Delta t )</td>
<td>1.2 s</td>
</tr>
<tr>
<td>Car free-flow speed</td>
<td>( u )</td>
<td>80 km/hr</td>
</tr>
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<td>Congestion wave speed</td>
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<tr>
<td>Jam density</td>
<td>( \kappa )</td>
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</tr>
<tr>
<td>Optimal density</td>
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</tr>
<tr>
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<td>( Q )</td>
<td>2400 veh/h</td>
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<tr>
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<td>75</td>
</tr>
<tr>
<td>Number of lanes</td>
<td>( n )</td>
<td>1 or 2</td>
</tr>
<tr>
<td>Length of each block</td>
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<td>160 m</td>
</tr>
<tr>
<td>Cycle length</td>
<td>( c )</td>
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<td>Bus free-flow speed</td>
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<tr>
<td>Probability of stopping</td>
<td>( p_s )</td>
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<tr>
<td>Dwell time</td>
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</tr>
<tr>
<td>Headway</td>
<td>( h )</td>
<td>varies</td>
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Table 1: Variables, notations, and default values for experiments
will have spilled all the way back to the entrance of the corridor.

Output results are normalized to give more generality in our results since all parameters become dimensionless. Following Laval and Castrillon (2015) we set the maximum flow \( Q = 1 \) and jam density \( \kappa = 1 \), and define the dimensionless parameters as:

\[
\lambda = \frac{l}{l^*}, \quad \rho = \frac{r}{g} \quad \tau = \frac{o}{c} \quad \gamma = \frac{h}{g}, \quad \alpha = \frac{v}{u} \tag{3}
\]

where \( \lambda \) represents block length measured in units of the critical block length, \( l^* = g(1/u + 1/w) \) and \( \rho \) is the red to green ratio. It is worth mentioning that the critical block length is the height of the “influence triangle” to be described momentarily on Fig. 9 to explain the short-block effect; see Laval and Castrillon (2015) for more details. Notice that \( \tau \) and \( \alpha \) will vary between 0 and 1, but the remaining parameters are simply positive real numbers.

**Experiment 1 - The effect of buses and topological parameters**

The purpose of this experiment is to examine the effect of bus operations on homogeneous corridors by varying block lengths, and signal parameters. Twenty-seven roadway cases are considered, which are obtained by combining three different \( \lambda \) values (0.5, 1, 1.5), \( \rho \) values (0.5, 1.0, 1.5), and \( \tau \) values (0.0, 0.33, 0.66). Only nine cases are shown on each diagram on Figure 3 for clarity, but the following findings are general:

F(a) The MFD for the "NO BUS" case is an upper bound to the MFDs when buses are introduced. This upper bound is tight during regions of low flow (but usually not tight during regions of moderate to high flow), which means that buses no longer constrain traffic.

F(b) The MFD flow is monotonically decreasing as bus headway increases for all experiments, as expected.

F(c) The effect of bus headway on the MFD varies greatly depending on corridor configuration. As \( \rho \) increases, the flow decreases for all bus headways, as expected. As \( \lambda \) and \( \tau \) changes, the MFD shape changes drastically.

Notice that although the bus maximum speed \( v \) is kept constant in all the experiments, one can see from the figure that the MFD free-flow speed in the figure seems different in each case, and that it explains most of the differences with respect to the no-bus MFD. As we will see in the next section, this corresponds to the average bus free-flow speed, \( \tilde{v} \).
Figure 3: Experiment 1: Each diagram represents a different corridor configuration by varying corridor parameters $\lambda$, $\rho$, and $\tau$. For every diagram, there are 8 different color-coded MFD curves, each with a different mean dimensionless headway, $\gamma$. Recall that to obtain a headway with dimensions, eqn. (3) must be used, i.e. $h = \gamma g$. Simulation parameters include: $g = 36\text{ s}$, $v = 40\text{ km/h}$, $p = 1/3$, $d = 20\text{ s}$, $b = 75$ blocks, $n = 1$ lanes.
**Experiment 2: The effect of corridor length**

This experiment examines how the MFD changes as the corridor length increases. The results on Figure 4(a) suggest the following:

F(d) As the number of blocks increases, the shape of the MFD in the left vicinity of capacity converges towards a straight line, similar to the moving bottleneck line on unsignalized corridors.

F(e) This convergence is almost independent of bus headway.

The time-space diagram on Figure 4(b) provides a simplified explanation for this behavior, by ignoring the effects of traffic lights. When buses enter or exit the corridor, traffic states A and C are induced at the beginning and end of the corridor, whose area is independent of the corridor length. It follows that for long corridors the effect of these areas vanishes, and traffic states U and D of the moving bottleneck model dominate. Notice that a similar effect was found in the case of trucks on two-lane rural roads Laval (2006a).

**Experiment 3: The effect of bus operational parameters**

This experiment examines the effects of operational parameters \( v, p \) and \( d \) while keeping bus average free-flow speeds approximately constant. Three different roadway cases are shown on Figure 5. Each case is depicted by two diagrams: the MFD and the average bus speed vs. density. Average bus speeds on the y-axis are normalized to the maximum car speed \( u = 80 \text{ km/h} \).

For each case, four different simulation runs are chosen with similar bus average free-flow speeds, but with varying bus operational parameters. For instance, on the first case (top two diagrams), the normalized average free-flow speed is close to 0.12. Notice that all curves converge to this value when densities are close to zero. The results on Figure 5 indicate the following:

F(f) As bus input parameters differ, bus average speeds also differ during free-flow and capacity conditions; during congested conditions they converge.

F(g) The bus free-flow average speed appears to be essential when determining the MFD. The left column of the figure suggests that the average bus free-flow speed might be the most important bus input operational parameter to consider even as other bus operational parameters vary.

The objective now is to develop an analytical method able to capture the insights revealed in this section. This is shown next.
Figure 4: (a) Experiment 2: Simulation runs with varying number of blocks. The input parameters are: $\lambda = 1.0$, $\rho = 0.5$, $\tau = 0$, $g = 30 \text{ s}$, $v = 30 \text{ km/h}$, $p = 1/3$, $d = 20 \text{ s}$, $n = 2$ lanes; recall that dimensional variables can be obtained with eqn. $(3)$. (b) Kinematic wave time-space diagram example showing the boundary effects of buses.
Figure 5: Experiment 3: Three different roadway configurations, with four different simulation runs each. For a given configuration, all runs have similar bus average free-flow speeds but varying bus operational parameters. For instance, for the top case, the input parameters \((v, p, d)\) are \((20 \text{ km/h}, 0, 0 \text{ s})\), \((40 \text{ km/h}, 0.5, 18 \text{ s})\), \((60 \text{ km/h}, 0.5, 24 \text{ s})\), \((80 \text{ km/h}, 1, 12 \text{ s})\).
5. Modified method of cuts

In this section we develop the mMOC using the insight learned from the simulation experiments to include buses as moving bottlenecks. From finding $F(a)$ we observe that the MFD for the "NO BUS" case is an upper bound to the MFD when buses are introduced. However, this upper bound is not tight for regions of moderate to high flow. The new method consists in adding two additional cuts to provide a tighter upper bound to the MFD; see Figure 6(b). The moving bottleneck cut is based on the endogenous determination of the (i) average bus free-flow speed, which captures the effects of operational parameters, and (ii) its maximum passing rate, which depends on signal settings. A second cut is needed because buses trigger an additional capacity reduction similar to the "short block" effect identified in Laval and Castrillon (2015). These additional cuts are presented below.

The proposed method extends Xie et al. (2013) by including the capacity reduction effect, the endogenous computation of average bus speed and the signal timing dependency of bus maximum passing rates.

5.1. Moving bottleneck cut

Finding $F(d)$ gives the strong indication that the moving bottleneck effect is indeed present in signalized corridors, and that it can be predicted by
moving bottleneck theory. In order to test this hypothesis, we find the moving bottleneck "line" using the VT framework. In the following two subsections, we propose methods to estimate both the bus average speed and its maximum passing rate.

5.1.1. Bus average free-flow speed

Recall that the bus travels along a homogeneous corridor at free-flow speed \( v \) and stops at red signals or at bus stops with probability \( p \). The time to board/alight passengers is \( d \), deterministic. The total distance traveled on a particular block is equal to the length of the block \( l \), while the time elapsed on the block is equal to the free-flow travel time \( l/v \) plus the delay from bus stops, \( \Delta_s \), and the delay if this signal is red, \( \Delta_r \). These delays are random variables and will be approximated by their expected value, \( E[\cdot] \). Accordin to renewal theory, the long-term average bus free-flow speed is given by:

\[
\tilde{v} = \frac{l}{l/v + E[\Delta_s] + E[\Delta_r]} \tag{4}
\]

where \( E[\Delta_s] = pd \) since the number of blocks that the bus travels before stopping can be modeled as a geometric process with mean \( 1/p \). To find \( E[\Delta_r] \) we neglect the effect of bus stops on whether or not the bus will stop at the following red light. In such case, we can use the result in Daganzo and Geroliminis (2008), which states that the number of blocks an observer travels before hitting a red phase is given by:

\[
N_{\text{max}} = 1 + \max\{N : [N(l/v - o + c)] \ mod \ c \leq g/c\} \tag{5}
\]

where \( o \) is the offset and \( c \) is the cycle length. The time it waits at a traffic signal is \( R = c - [N_{\text{max}}(l/v - o + c)] \ mod \ c \). The approximated average wait time per block is therefore \( E[\Delta_r] = R/N_{\text{max}} \).

In the sequel we use the following dimensionless version of (4) is:

\[
\alpha = \frac{\tilde{v}}{u} = \frac{l/u}{l/v + pd + R/N_{\text{max}}} \tag{6}
\]

We have tested the quality of this approximation method. We generated 1,000 values for each of the parameters \( \rho, \lambda, \tau, p, \) and \( d \). For each set of

\[\text{Notice that obtaining a general expression for } N_{\text{max}} \text{ that accounts for dwelling is difficult because traffic signal settings are deterministic and therefore renewal theory cannot be applied.}\]
parameter values, the average bus speed was estimated using (6) and also using a simulated trajectory with the specified parameters over a very long corridor. Remarkably, the proposed approximation is unbiased, as shown in Fig. 7, which means that we have correct estimates of the expected values in (4), as needed.

5.1.2. Bus maximum passing rate

Maximum passing rates in a VT time-space network are challenging to calculate analytically because of the intricate interactions between signals and buses. In order to overcome this issue we separate the corridor into two sets of lanes: one mixed car-bus lane containing all buses, and the remaining car-only lanes. Here, we are assuming that at any given location there is no more than one bus on all lanes, which is reasonable because buses usually operate and make stops on the outside lane.

The advantage of separating the lanes is that for the mixed lane, the
maximum passing rate for the moving bottleneck is zero, since no cars can
overtake buses on that lane. Also, the maximum passing rate on the car-only
lane, \( R_{\text{car-only}}(\tilde{v}) \), can be readily obtained using the moving observer formula
(Newell, 1998). This can be accomplished graphically by placing a line with
slope \( \tilde{v} \) tangent to the MFD curve and calculating its y-intercept; see Figure
8. The final maximum passing rate for all lanes is therefore:

\[
R(\tilde{v}) = \frac{n - 1}{n} R_{\text{car-only}}(\tilde{v})
\]  

where \( n \) is the total number of lanes. Notice that for piecewise linear MFDs
chances are that this line will pass by a discontinuity point such as point "1"
on Figure 8.

5.2. The bus short-block cut

Estimating the capacity of a homogeneous corridor with buses becomes
challenging because of combined effects of running buses, bus stops and sig-
nalized intersections. Even in the absence of buses, Laval and Castrillon
(2015) describe how short blocks make it increasingly difficult to calculate
minimum paths on a VT network. The main problem is with the stationary
cut (with zero average speed) since its related minimum path is not straight
but takes “detours” to neighboring intersections; see Fig. 9a. As mentioned
in that reference, this happens whenever a red phase falls in the influence
triangle depicted in the figure. To date, analytical approximations to this
phenomenon have not been found. Recall, however, that this phenomenon
Figure 9: (a) Minimum paths are altered by short-blocks in the form of detours, (b) buses alter minimum paths similar to short-blocks.
is automatically captured within numerical solution methods such as the
simulation model presented here (in appendix), or VT networks.

Buses exacerbate the short-block effect by providing more opportunities
for detours due to both the slower travel speed and the stops; see Fig. 9b.
To overcome the problem of analytical intractability of this “bus short-block
cut”, a regression model is proposed.

The regression is based on the simulation results of a single mixed car-bus
lane. The model includes the topological dimensionless parameters, \( \lambda \), \( \rho \) and
\( \tau \) and the dimensionless bus parameters, \( \gamma \), \( \alpha \). Notice that bus parameters
such as the bus free-flow speed \( v \), the stop probability \( p \) and the dwell time
\( d \) are not necessary as long as \( \alpha \) is known; see finding F(g).

In order to develop the model, 1000 different training runs and 500 testing
runs are saved and the maximum capacity is obtained for each one. For each
simulation run, random variables are generated for each of the parameters.
Four of the parameters are uniformly distributed: \( \lambda \epsilon \{0.25, 2.25\} \), \( \rho \epsilon \{0, 2\} \),
\( \tau \epsilon \{0, 1\} \), and \( \gamma \epsilon \{0, 24\} \). Notice that the average speed of buses cannot be
manipulated in the simulation as it is a resulting function of the network
parameters, as well as the bus free-flow speed \( v \), the stop probability \( p \) and
the dwell time \( d \). Therefore, bus input parameters are uniformly distributed
with ranges \( p \epsilon \{0, 1\} \) and \( d \epsilon \{0, 60\} \) sec, while \( v \) can only take four discrete
values \( \{20, 40, 60, 80\} \) km/h. The final regression equation is obtained by
adding relevant variables and keeping the significant ones at the 0.001 level:

\[
Q_{\text{mixed}} = a_1 \alpha - a_2 \alpha^2 + a_3 \alpha^3 + a_4 \lambda - a_5 \rho \alpha + \log(\gamma a_6)
\]  

(8)

where \( a_1 = 2.781282 \), \( a_2 = 4.328240 \), \( a_3 = 2.589717 \), \( a_4 = 0.032333 \), \( a_5 =
0.544471 \), and \( a_6 = 0.034573 \).

The \( R^2 \) value for the training data set is 0.98. For the test data, the mean
absolute error is 0.039 and the root mean square error is 0.0464 (recall that
capacity ranges from 0 to 1). Notice that the offset \( \tau \) turned out to be not
significant, and the model only requires the remaining four parameters.

The model suggests that \( \alpha \) has a cubic polynomial effect on capacity as
well as an interactive effect with \( \rho \). Also, \( \lambda \) has a positive linear effect and \( \gamma \)
has a positive logarithmic effect on capacity. The marginal effect of the input
parameters is given on the first row of Table 2. The variable \( \rho \) has a negative
marginal effect, which is expected since the red phase increases compared to
the green phase. Also, \( \lambda \) has a positive marginal effect which is expected
since the queues from bus stops have a larger block length before they spill
back to upstream intersections. Furthermore, γ has a positive marginal effect on capacity which is also expected since an increase in headway translates to a decrease in the number of buses on the corridor.

The last row of the table gives the elasticity of input parameters evaluated at their mean values. The results suggest that ρ and α are the most important parameters when determining capacity at the mean input values.

To estimate the capacity of all lanes, we obtain the capacity of the mixed lane from Equation 8 and the capacity of the car-only lanes is obtained from the method of cuts; see Figure 8. The final equation is:

\[ Q_{total} = \frac{1}{n} Q_{mixed} + \frac{n-1}{n} Q_{car\text{-}only} \]

which defines the bus short-block cut.

Finally, notice that the regression model in this section should be applicable to any facility regardless of the fundamental diagram because (i) the variables are dimensionless and (ii) the symmetry property of the kinematic wave model outlined in Laval and Chilukuri (2016).

### 5.3. Results

In section we investigate the accuracy of the proposed method relative to (i) the original MOC, and (ii) the traffic simulation described in the Appendix.

The results on Fig. 10 indicate that the original MOC (in green) provides an upper bound approximation to the simulation results (in black), which in many cases is not tight. However, when the proposed bus cuts are introduced (the bus moving bottleneck cut and the bus short-block cut shown in purple), the combined lower envelope improves the agreement with the

<table>
<thead>
<tr>
<th>( \frac{dQ_{mixed}}{dx} )</th>
<th>( x = \rho )</th>
<th>( x = \lambda )</th>
<th>( x = \gamma )</th>
<th>( x = \alpha )</th>
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<tr>
<td>(-a_5\alpha)</td>
<td>(a_4)</td>
<td>(a_6/\gamma)</td>
<td>(a_1 - 2a_2\alpha + 3a_3\alpha^2 - a_5\rho)</td>
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<td>1.229</td>
<td>11.924</td>
<td>0.224</td>
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<td>( x )-elasticity of ( Q_{mixed} )</td>
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<td>0.092</td>
<td>0.080</td>
<td>0.341</td>
</tr>
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</table>

Table 2: Marginal effect and elasticity of inputs
simulation results. Notice that including the moving bottleneck cut alone is not sufficient in some cases, but the short-block cut is necessary to provide a good approximation to the capacity. Conversely, the two write-most cases on the top row of the figure indicate that on some network configurations the proposed cuts are not necessary. This happens when $\bar{v}$ is greater than the speed of the free-flow cut near capacity; this can be explained by the duality of traffic lights and moving bottlenecks revealed in Laval (2006b).

In closing, our results suggest that the proposed mMOC method provides an accurate approximation to the MFD, and that the main parameter is the bus average free-flow speed, $\bar{v}$, which dictates the magnitude of both the moving bottleneck and bus short-block cuts.

6. Discussion

Our findings suggest that the interaction between buses, block lengths and signal parameters is essential when determining the MFD of an urban arterial corridor where buses operate. Notably, our results suggest that these complex interactions can be boiled down to a single parameter, the bus average free-flow speed. This paper provides both a method to estimate it based on observable parameters, and a method to derive its associated impacts on the MFD. We found that the two major effects that buses have on a corridor are the moving bottleneck effect and the reduction in capacity due to the "short block" effect.

This research constitutes an initial step in understanding the effects of buses on the MFD. Although homogenous corridors are a simplification of real life corridors, they help to understand the interactive effects between traffic signal bands (red and green waves) and bus operations. The model proposed here will probably not be directly applicable to heterogeneous corridors but (i) the methodology to calculate, endogenously, the bus average travel speed in free flow, and (ii) the insights obtained from the moving bottleneck effect and the capacity reduction, should be. A follow-up paper is being developed to apply what was learned here to heterogeneous corridors.

Future research can focus on extending homogeneous corridors to heterogeneous corridors with varying block lengths and signal parameters as well as traffic networks with origins and destinations and routing for each vehicle. Networks introduce local heterogeneities whose traffic properties can be obtained from averaging local corridor properties with varying topologies, signal timings and traffic demand. For instance, if one considers a given $\rho_{east}$
Figure 10: Comparisons between simulation results, method of cuts (no buses) and bus cuts. The same $\rho$, $\lambda$ are used from Experiment 1, but only one $\gamma$, $\tau$ and $n$ value are used for each roadway case. The remaining parameters are: $b = 75$ blocks, $g = 36$ s, $v = 40$ km/h, $p = 1/3$, $d = 20$ s.
(red/green ratio) on the eastbound direction, this automatically assumes a
\[ \rho_{\text{west}} = 1 - \rho_{\text{east}} \]
on the westbound direction. Therefore, we now have two cor-
ridors with varying properties. Other topics are to examine passenger flow
capacity, dedicated bus lanes, or how demand-driven transit operations such
as bus-bunching interact with the MFD of a corridor. The authors are cur-
rently investigating these topics.

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Appendix: Simulation model

This appendix describes the numerical model to simulate the experiments in this paper, which are limited to arterial corridors (not networks).

Cellular Automata traffic simulation

We use a simulation model that gives the exact solution of the Kinematic Wave model Lighthill and Whitham (1955); Richards (1956) with a triangular fundamental diagram, using cellular automaton (CA) Daganzo (2006). Time and space are discretized by $\Delta t$ and $\Delta x$, where each vehicle occupies one cell. A variable with a dimensionless quantity is described with a "hat", e.g. $\hat{x} = x/\Delta x$. For a single link, the dimensionless position is given by:

$$\hat{x}(i, \hat{t} + 1) = \min\{\hat{x}(i, \hat{t}) + u/w, \hat{x}(i - 1, \hat{t}) - 1\} \tag{10}$$

Where $\hat{x}$ is the dimensionless position measured in units of the jam spacing $\kappa$, $i$ is the vehicle number, $\hat{t}$ is dimensionless time measured in units of $1/(\kappa w)$, and $u/w$ is an integer.
Cars travel at free-flow speed 4 cells/time unit which corresponds to 80 km/hr when \( \Delta x = 6.67 \) and \( \Delta t = 1.2\, \text{sec} \). These parameters are similar to the range of values used in the field. Flow and density averages for a time aggregation period \( \tau \) and a lane-distance aggregation area \( X \) are calculated by using Edie’s generalized definitions Edie (1963):

\[
q = \frac{D}{\tau X}, \quad (11a)
\]

\[
k = \frac{T}{\tau X}, \quad (11b)
\]

where \( q \) is the flow, \( k \) is the density, \( D \) is the sum of the distance covered by all vehicles, and \( \tau \) is the sum of the time spent by all vehicles on the system.

**Lane-changing model**

We implemented the lane-changing rules in Nagel et al. (1998), which consist on determining whether there is an incentive to change lanes, and then looking for sufficient gap on the target lane before making the change. The incentive to change lanes has one important parameter called the “look ahead” value, which is the number of cells in front to look for another vehicle. If there happens to be a vehicle within the look ahead value, then the velocity of the vehicle ahead is compared to its own velocity. If the velocity is slower than its own, then the incentive to change lanes becomes active and it looks for a large enough empty gap on the target lane to make the change. The ”look ahead” value used is 6 cells and the lane changing gap is 5 cells where a vehicle looks at 4 cells behind and 1 lane in front of the current cell location. These values were calibrated to ensure that the model replicates the single-pipe moving bottleneck model described in figure 1.

This simulation model has been implemented as a network simulation to include traffic intersections, turns, routing and lane-changing. A beta version of the software can be accessed online \(^2\). The simulation software has user-friendly controls, a real-time visualization component to see how vehicles interact in the system, and an MFD output component.

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\(^2\)MFD multimodal simulation: http://felipecastillon.github.io/mfd_simulation_bus/