A macroscopic theory of two-lane rural roads

Jorge A. Laval [a,*]

[a] *Laboratoire Ingénierie Circulation Transport LICIT (INRETS/ENTPE), Rue Maurice Audin, 69518 Vaulx en Velin Cedex, France.*

Abstract

A macroscopic theory for predicting the operation on two-lane, two-way roads is proposed. In this theory, the interaction between fast and slow vehicles obeys Newell's kinematic wave theory of moving bottlenecks. Calibration is not required as all parameters are fully observable. Closed-form expressions for the capacity, average speed, percent time spent following and overtaking rates are proposed and the biases of current practice are identified. Comparisons between the proposed theory and empirical data are also included.

Key words:
Two-lane rural roads, kinematic wave model, moving bottlenecks.

1 Introduction

Theories for predicting traffic operations in rural roads have a long history. The subject received much attention around the 60' and 70' (see Daganzo, 1975, and references therein) but surprisingly little attention lately. This is unfortunate because earlier results have not been proven successful and the state-of-the-art fully relies on microsimulation models.

Earlier results are only applicable to very light traffic since they are based on steady-state queueing models (eg, M/M/1), which neglect physical queues. Moreover, it was always assumed that traffic in the opposing direction has an exogenously defined probability distribution (typically Poisson). This assumption makes the problem ill-defined because it requires knowing in advance precisely what we are looking for; ie, it neglects the *recursive nature* of the

* Tel. : +33 4 72 04 70 50, fax : +33 4 72 04 77 12
Email address: `jorge.laval@entpe.fr` (Jorge A. Laval).

problem: traffic in each direction is affected by opposing flow in the same way. To tackle this complexity several microsimulation software packages have been developed (Brodin and Carlsson, 1996; Hoban et al., 1991; McLean, 1989), and are used among researchers and practitioners alike. In fact, the methodology proposed in the Highway Capacity Manual (HCM, 2000) is based on one of these models. Unfortunately, these models have been found to disagree with empirical data (Dixon et al., 2002; Hegeman, 2004), despite the numerous parameters and input data needed. Cellular automata models have also been suggested (Fouladvand, 2000; Simon and Gutowitz, 1998), but have not been validated either.

To the authors' knowledge models based on the kinematic wave theory of Lighthill and Whitham (1955) and Richards (1956) have not been developed. This is surprising since this is the simplest model that correctly predicts traffic dynamics on a single lane. Furthermore, Newell's kinematic wave theory of moving bottlenecks (Newell, 1998) allows us to predict the effects of a slow vehicle (SV) on a traffic stream, which has been shown to explain several phenomena on multi-lane unidirectional traffic (Laval, 2005a,b; Laval et al., 2005; Laval and Daganzo, 2006). To fill this void, this paper proposes a model based on moving bottleneck theory that incorporates the recursive nature of the problem explicitly. We obtain closed-form approximations of the main performance measures for rural roads. Towards this end, §2 formulates the problem, §3 presents the model, and §4 derives relevant performance measures, which are used to validate the model with field data. Finally, a brief discussion is included in §5.

2 Problem formulation

Consider a long two-lane, two-way highway where no-passing zones are neglectable. Each lane obeys a triangular fundamental diagram defined by its free-flow speed u, wave speed $-w$ and capacity Q; see Fig 1a. For one direction of travel the input demand corresponds to state A in the figure, where a small (time-mean) proportion of SVs, r, travels at a free-flow speed $v < u$. Variables on the opposite direction are denoted by a prime (eg, $q_{A'}, r'$, etc).

We are interested in describing congested situations where platoons form due to the presence of SVs; ie, when SVs hold back a queue upstream while free-flow conditions are observed downstream (until the next platoon). Let D be the long-term average free-flow state observed downstream of a SV when it holds back a queue, and let U be the state of this queue. Notice that this definition implies that the overtaking process will be described in its mean value over time, including periods where passing is impossible and periods where passing takes place at some maximum rate. Notice too that q_D will be

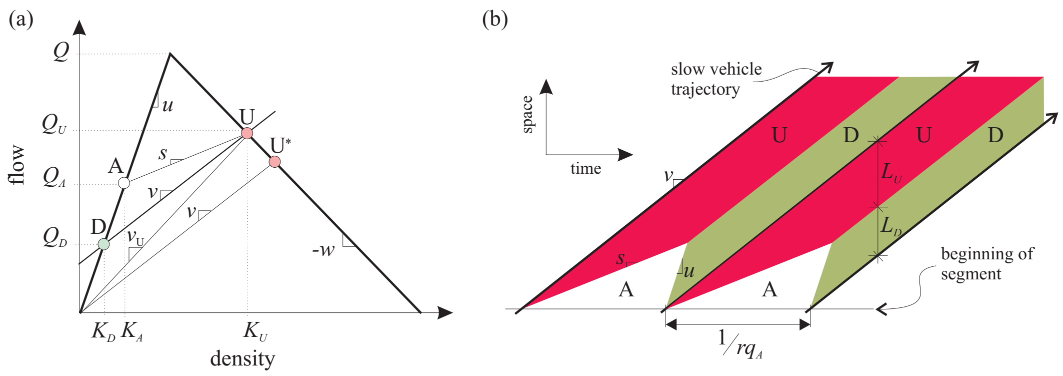

Fig. 1. Problem formulation in the (a) fundamental diagram and in the (b) time - space diagram.

endogenously defined in the model, es explained next.

The flow corresponding to a given traffic state S is denoted Q_S, its speed v_S and its density K_S. To simplify the exposition we use adimensional flows $q_S = Q_S/Q$ and density in units of pace $k_S = K_S/Q$; note that $q_S = k_S v_S$ still holds. For clarity in notation, the only exception to this notation rule is q_{U^*} (see Fig. 1a), which will be denoted c, ie

$$c = \frac{(u+w)v}{(v+w)u}. \tag{1}$$

Notice that using adimensional flows together with (1) will enable us to express q_D in terms of a single exogenous parameter, c, which is fully observable.

According to moving bottleneck theory (Newell, 1998), states D and U are connected by a straight line of slope v; see Fig. 1a. Therefore, if D is known one can obtain U by intersecting this line with the congested branch of the fundamental diagram, ie

$$k_U = \frac{c}{v} - \frac{\bar{c}}{w} q_D, \qquad q_U = c + \bar{c} q_D, \qquad v_U = q_U/k_U, \tag{2}$$

where we have defined the function "overbar" that gives the complement of a dimensionless variable; eg, $\bar{c} = 1 - c$.

The time-space diagram for one direction corresponding to our problem is presented in Fig. 1b. The length of the queue generated by a SV, L_U, and the length of the free-flow state upstream until the next SV, L_D, are determined at the beginning of the road section. These quantities are a consequence of the collision between the shock waves from the transitions A→U, with slope s, and A→D, with slope u; ie,

$$L_U = \frac{(v-s)u}{(u-s)rq_A}, \qquad L_D = \frac{(u-v)s}{(u-s)rq_A}. \tag{3}$$

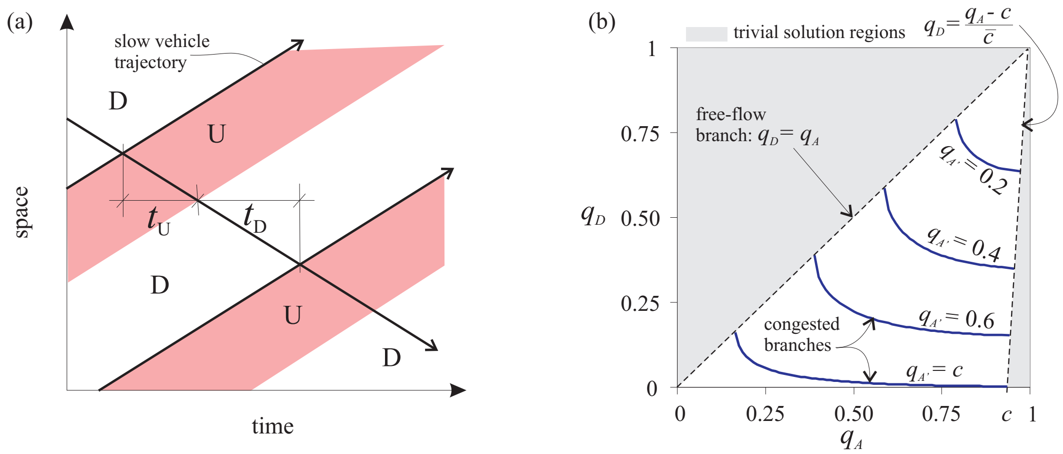

Fig. 2. (a) A SV crossing two SVs on the opposite direction; (b) downstream state as predicted by eqns. (7)-(8).

We are interested in the non-trivial cases $0 \leq s \leq v$, or equivalently

$$q_D \leq q_A \leq q_U, \tag{4}$$

as in Fig 1a. When (4) is not satisfied trivial solutions are adopted: if $q_A < q_D$ there is no congestion, $L_U = 0$ and all fast vehicles travel at free-flow speed; when $q_A > q_U$ a queue propagates upstream of the entrance and the entire segment operates in state U.

3 Model formulation

It follows from the preceding section that the only unknown is q_D. Our goal is defining q_D endogenously in the model, rather than assuming exogenous values, as customary. Towards this end, we utilize the recursive nature of the problem; ie, that there exists a physical process, $f(\cdot)$, that describes q_D as a function of relevant variables, and that this process applies to both directions following the same rules. To identify a well-defined general relation among variables, one can use the Buckingham $\pi-$theorem (Buckingham, 1914), which suggests that our problem can be stated as the following system of equations

$$q_D = \eta f(q_{A'}, q_{D'}, c), \tag{5a}$$
$$q_{D'} = \eta' f(q_A, q_D, c), \tag{5b}$$

where η and η' are adimensional constants. This paper proposes a functional form for $f(\cdot)$ based on the assumption that overtaking is possible only when traffic in the opposite direction is in free-flow. In these circumstances $f(\cdot)$ can be interpreted as the proportion of time that a SV sees state D in the opposing direction, and η as the mean overtaking dimensionless flow.

If t_D and t_U are the time intervals where a SV sees the opposing direction in state D and U, respectively, it follows that the proportion of time passing is possible is $t_D/(t_D+t_U) = L_D/(L_D+L_U)$; see Fig. 2a. Using (3) and eliminating s by means of the shock condition $s = (q_A - q_U)/(k_A - k_U)$, yields after manipulation

$$q_D = \eta \frac{c + \bar{c} q_{D'} - q_{A'}}{c \bar{q}_{D'}}, \tag{6a}$$

$$q_{D'} = \eta' \frac{c + \bar{c} q_D - q_A}{c \bar{q}_D}. \tag{6b}$$

This is a quadratic system of equations, whose meaningful solution is

$$q_D(q_A, q_{A'}) = \frac{c_0 + (\eta' q_A - \eta q_{A'})c - \sqrt{c_1 + c_2 q_A + c_3 q_{A'} + c^2(\eta' q_A - \eta q_{A'})^2}}{2(c + \eta' \bar{c})c}. \tag{7}$$

where $c_0 = c^2(\eta + \bar{\eta}') - \bar{c}^2 \eta\eta'$, $c_1 = c^4(\bar{\eta}^2 + \bar{\eta}'^2 - 1) - \eta\eta'[2c^2(\bar{c}^2(\eta + \eta') + 2c\bar{c} + 1) - \bar{c}^4 \eta\eta']$, $c_2 = 2(\eta\eta' + 2\eta\bar{\eta}' c + \bar{\eta}'\bar{\eta} c^2)c\eta'$ and $c_3 = 2(\eta\eta' + 2\eta'\bar{\eta} c + \bar{\eta}'\bar{\eta} c^2)c\eta$. The solution for $q_{D'}$ is identical to (7) but interchanging the primes. To ensure that (7) remains real in the region defined by (4), parameters η and η' are chosen such that the square root term vanishes at $q_D = q_A$; this gives

$$\eta = \frac{c + \bar{c} q_A}{c + \bar{c} q_{A'}} c \bar{q}_{A'}, \qquad \text{and} \qquad \eta' = \frac{c + \bar{c} q_{A'}}{c + \bar{c} q_A} c \bar{q}_A. \tag{8}$$

Notice that for typical conditions $c \approx 1$ and thus $\eta \approx 1 - q_{A'}$. Thus, (8) can be interpreted, roughly, as the mean proportion of time a void is found in the opposite direction, which is consistent with our previous interpretation. Notice that (8) avoids introducing additional exogenous parameters that would require calibration and makes our model a function of c, q_A and $q_{A'}$ only.

Figure 2b shows the predictions of (7)-(8) as a function of q_A for several values of $q_{A'}$. Recall that this model applies only when (4) is satisfied; otherwise trivial solutions are adopted, as explained earlier. It is straightforward to show that this condition defines a feasible region for (7)-(8) of triangular shape (white region in Fig. 2b) with boundaries $q_D = 0$, $q_D = q_A$ and $q_D = (q_A - c)/\bar{c}$. The line $q_D = q_A$ can be interpreted as an *uncongested branch* of the $q_D - q_A$ diagram while a given opposing flow $q_{A'}$ defines a particular *congested branch.* As expected, q_D is a decreasing function of opposing demand $q_{A'}$. Interestingly, inside a congested branch, q_D is also a decreasing function of same-direction demand q_A. This is precisely the phenomenon we wanted to capture: as q_A increases $q_{D'}$ decreases and the length of the queue in the opposing direction will increase. In turn, this will decrease passing opportunities in the same direction and thus q_D will decrease. Finally, notice that overtaking may become impossible only when $q_{A'} \geq c$; eg, if $\{q_A = c, q_{A'} = c\}$ then $q_D = 0$.

4 Performance measures

In this section we define commonly used measures of performance (MOP) and use them to validate the model with empirical data.

4.1 Percent time spent following

The HCM 2000 defines the percent time spent following, PTSF, as the average percentage of travel time vehicles travel in platoons behind slower vehicles because of the inability to pass. Unfortunately, the surrogate estimate adopted by researchers is the proportion of headways less than some critical value, t_c, as observed at a fixed location. This estimation procedure is ill-defined because a headway of, say three seconds, maybe observed both in free-flow and congestion. As a consequence, the PTSF can be highly overestimated in practice.

To see the bias introduced by this surrogate estimate, let $PTSF_\phi$ be the PTSF measured along a vehicle trajectory, and $PTSF_x$ the PTSF measured at location x. Along a vehicle trajectory the time spent following a single SV is $t_U = L_U/(v_U - v)$ and the time in free-flow before reaching the queue of the next SV is $t_D = L_D/(u - v)$. At a fixed location one would see that between two consecutive SVs vehicles pass queueing during the first $t_U = L_U/v$ time units and in free-flow during the following $t_D = L_D/v$ time units. In both cases the PTSF is given by $t_U/(t_U + t_D)$, which after manipulation gives

$$PTSF_\phi = \frac{(w - q_D v\bar{c}/c)(q_A - q_D)}{wq_A - (v + w)\bar{c}q_D}, \tag{9}$$

$$PTSF_x = \frac{q_A - q_D}{c\bar{q_D}}. \tag{10}$$

The differences between these two equations can be seen in Fig. 3a-b for different values of q_D. It is apparent how $PTSF_\phi \geq PTSF_x$ under all conditions. Notice that this result is true in general (ie, independent of the proposed model). Part b of the figure includes empirical data from Dixon et al. (2002), where a critical value $t_c = 3$ sec was assumed. It can be seen how the empirical values are well above $PTSF_x$, which illustrates the bias introduced by current field PTFS estimates.

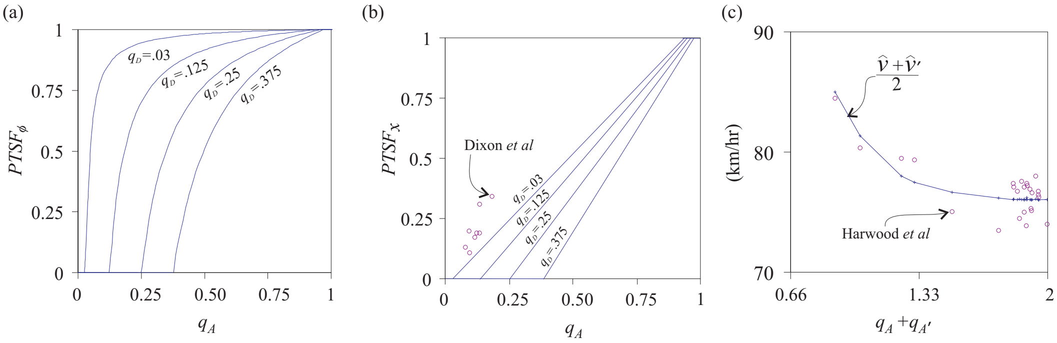

Fig. 3. Performance measures: (a) $PTSF_\phi$ as a function of q_A for several values of q_D; (b) $PTSF_x$ as a function of q_A for several values of q_D and for $t_c = 3$ sec; and (c) 2-lane average of the space-mean speed.

4.2 Average speed

The space-mean speed, $\hat{v}$, can be computed as $(L_U k_U v_U + L_D k_D u)/(L_U k_U + L_D k_D)$, which gives

$$\hat{v} = \frac{v q_A}{q_A - (1 - v/u) q_D}. \tag{11}$$

As expected, $\hat{v} \to u$ when $q_D \to q_A$, and $\hat{v} \to v$ when $q_D \to 0$. Fig. 3c shows the predictions of (11) for the field data of tables 22 and 29 in Harwood et al. (1999), which presents the mean speed for both directions of travel combined, ie, $(\hat{v} + \hat{v}')/2$. As shown in the figure, using the reported speed limit $u = 85$ km/hr and assuming $w = 15$ km/hr, $Q = 1500$ vph and $v = 75$ km/hr, gives an adequate fit to this empirical data.

4.3 Overtaking rate

To obtain the number of overtakings per km-hr, Φ, we note that the passing rate along a SV trajectory is $R = q_D(1 - v/u)$ and that SVs are $L = v/rq_A$ km apart. Therefore, $\Phi = R/L$, which depends on r. To avoid r-dependencies we will use:

$$\Phi/r = (\frac{1}{v} - \frac{1}{u}) q_A q_D. \tag{12}$$

Figure 4a shows the predictions of model (7)-(8)-(12) assuming $u = 110$ km/hr, $w = 17$ km/hr, $Q = 1700$ vph, $v = 75$ km/hr, compared to the field data of Fig. 2a in Enberg and Pursula (1997). This reference reports 15-min overtaking rates for one direction, the corresponding 15-min flow on that direction and the aggregate range of truck proportion during the measurement period (we use the mean value $r = 0.06$). It also reports the extreme opposing

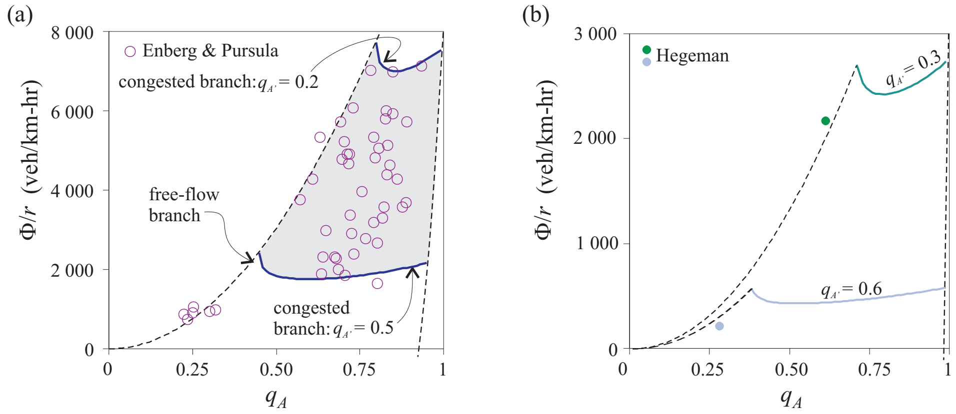

Fig. 4. Predicted and observed overtaking rate.

flow values, $q_{A'} = 0.2$ and $q_{A'} = 0.5$, which reduces the feasible region to the gray area in Fig. 4a. Reassuringly, the figure shows that almost all data points lay within the feasible region predicted by the model.

Figure 4b shows the predictions of the model assuming $w = 17$ km/hr and $Q = 1700$ vph, for the field data in Hegeman (2004). This reference reports 3-hr aggregate overtaking rates and flows for both directions, together with $u = u' = 110$ km/hr, $v = 83$ km/hr, $v' = 87$ km/hr, $r = 0.072$ and $r' = 0.052$. Fig. 4b shows that the model predictions are in agreement with empirical data. It also reveals that although the flow in one direction is high, free-flow conditions prevailed during the measurement period in both directions. This is true because the data points lay close to the free-flow branches, and well below the congested branches imposed by opposing flow (solid curves in the figure).

5 Discussion

The consistency between the model predictions and empirical data is appealing, especially since a single observable parameter is needed and thus no calibration is necessary. This is convenient since further refinements such as multiple SV classes or SV speeds differentiated by direction would turn the model untractable, and the results may or may not improve significantly. The good fit also indicates that at a macroscopic level the complex real-life overtaking behavior may be well represented by the proposed simplified model; ie, overtaking is possible only when the opposing lane is in free-flow, and takes place at a rate proportional to the proportion of time a void is found in that lane.

With the exception of the overtaking rate, all the results in this paper are independent of the proportion of SVs, r. This appealing result is a consequence of L_U/L_D being independent of r; ie, that the proportion of time spent in each

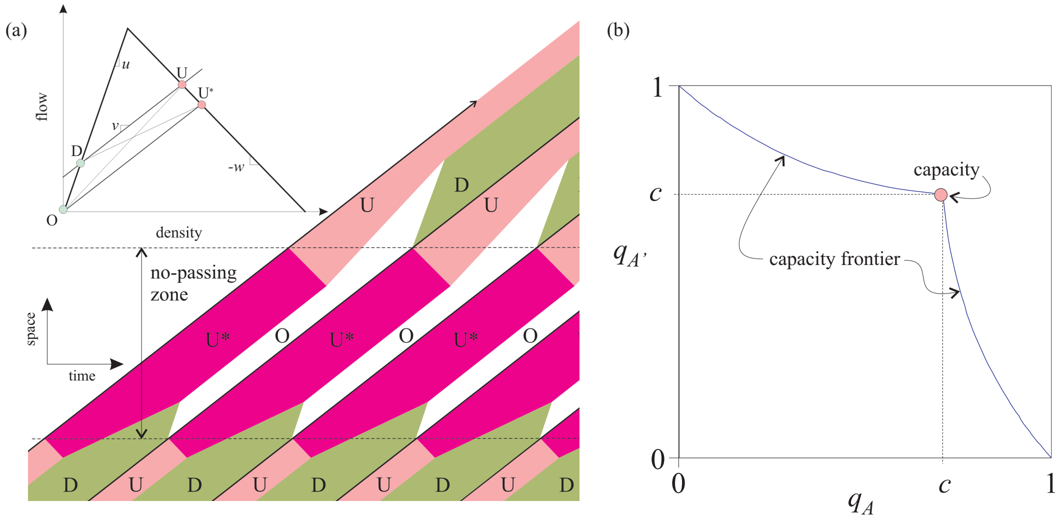

Fig. 5. (a) Time-space diagram showing the effects of no-passing zones; (b) the capacity frontier and the capacity of two-lane roads.

state U or D is independent of the number of SVs. This should be true for long roads, where one has the chance to overtake at least two SVs. An important implication is that the results are independent of stochastic fluctuations in SV arrivals; in the case of the overtaking rate, (12) still holds in terms of expected value.

The formulae and empirical data presented in this paper correspond to facilities with a small proportion of no-passing zones. In cases where no-passing zones cannot be neglected, MOPs can be estimated in two steps. One can show that away from these zones the values of L_U and L_D remain unchanged w.r.t. (3), provided that $q_D \leq c$; see Fig. 5a. This condition should always be satisfied except in rare circumstances. Therefore, one may calculate a certain MOP for passing and no-passing zones separately, and then take the average weighted by the corresponding space-proportions. For no-passing zones one just needs to set $q_D = 0$ in the relevant MOP.

Capacity can also be computed using the proposed model. Given q_A, the maximum opposing flow, $q^*_{A'}$, must satisfy the condition $q_A = q_U(q_A, q^*_{A'})$. A value bigger than $q^*_{A'}$ would induce queues in the direction of q_A that would propagate upstream of the entrance, and therefore the total throughput would be smaller than $q_A + q^*_{A'}$. One can show that

$$q^*_{A'} = \frac{(2 - c^2)\bar{c}q_A + \bar{c}c^2 - \sqrt{a_1 q_A^2 + a_2 q_A - a_3}}{2\bar{c}(c - \bar{c}^2 q_A)}, \tag{13}$$

where $a_1 = \bar{c}^2(c^4 - 4c^3 + 4c^2 + 4), a_2 = 2\bar{c}c^2(c^3 - 5c^2 + 8c - 2)$ and $a_3 = \bar{c}c^3(c^2 - 5c + 8)$. This equation defines the capacity frontier shown in Fig. 5b. Notice that total flow $q_A + q_{A'}$ is maximized at $\{q_A = c, q_{A'} = c\}$; ie, when demand is symmetric and equal to the flow in the queue of a SV and there is no overtaking. We conclude that capacity equals $2c$. This indicates that capacity of two-lane roads is not constant (as commonly assumed) but heavily dependent on the speed of SVs.

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References

Brodin, A., Carlsson, A., 1996. The VTI traffic simulation model. Tech. rep., Swedish National Road and Transport Research Institute, Linkping.

Buckingham, E., 1914. On physically similar systems; illustrations of the use of dimensional equations. Physics Reviews (4), 345–376.

Daganzo, C. F., 1975. Probabilistic structure of two-lane road traffic. Transportation Research Part B 9 (4), 339–343.

Dixon, M., Sarepali, S., Young, K., 2002. Field evaluation of highway capacity manual 2000 analysis procedures for two-lane highways. Transportation Research Record 1802, 125–132.

Enberg, A., Pursula, M., 1997. Traffic flow characteristics on a three-lane rural highway in finland results from empirical studies and simulations. Transportation Research Record 1572, 33–42.

Fouladvand, M. E., 2000. Reaction-diffusion models describing a two-lane traffic flow. Physics Reviews E 62 (5), 5940–5947.

Harwood, D., May, A., Anderson, I., Leiman, L., Archilla, A., 1999. Capacity and quality of service of two-lane highways. Tech. rep., NCHRP Project 3-55(3).

HCM, 2000. Highway Capacity Manual 2000. Transportation Research Board, National Academy of Sciences, Washington D.C.

Hegeman, G., 2004. Overtaking frequency. IEEE International Conference on Systems, Man and Cybernetics, 4017–4022.

Hoban, C., Shepherd, R., Fawcett, G., Robinson, G., 1991. A model for simulating traffic on two-lane rural roads: User guide and manual for TRARR version 3.2. Tech. rep., Australian Road Research Board, Victoria.

Laval, J. A., 2005a. Linking synchronized flow and kinematic wave theory. In: Khne, R., Poschel, T., Schadschneider, A., Schreckenberg, M., Wolf, D. (Eds.), Forthcoming in Traffic and Granular Flow'05. Springer.

Laval, J. A., 2005b. Stochastic processes of moving bottlenecks: Approximate formulas for highway capacity. Forthcoming in Transportation Research Records.

Laval, J. A., Cassidy, M. J., Daganzo, C. F., 2005. Impacts of lane changes at on-ramp bottlenecks: A theory and strategies to maximize capacity. In: Khne, R., Poschel, T., Schadschneider, A., Schreckenberg, M., Wolf, D. (Eds.), Forthcoming in Traffic and Granular Flow'05. Springer.

Laval, J. A., Daganzo, C. F., 2006. Lane-changing in traffic streams. Transportation Research Part B 40 (3), 251–264.

Lighthill, M. J., Whitham, G., 1955. On kinematic waves. I Flow movement in long rivers. II A theory of traffic flow on long crowded roads. Proc. Roy. Soc. 229 (A), 281–345.

McLean, J. R., 1989. Two-lane highway traffic operations (TWOPAS). In: Ashford, N., Bell, W. (Eds.), Transportation Studies. Vol. 11. Gordon and Breach Science Publishers, Amsterdam.

Newell, G. F., 1998. A moving bottleneck. Transportation Research Part B 32 (8), 531–537.

Richards, P. I., 1956. Shockwaves on the highway. Operations Research (4), 42–51.

Simon, P., Gutowitz, H., 1998. Cellular automaton model for bidirectional traffic. Physics Reviews E 57 (2), 2441–2444.