Effects of Geometric Design on Freeway Capacity: Impacts of Truck Lane Restrictions

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Abstract

This paper presents a framework for estimating analytical expressions for the capacity reductions caused by a a subset of vehicles forced to slow down at horizontal/vertical curves on multilane freeways. In each lane the underperforming stream is described in terms of its desired speed distribution (either discrete or continuous), and it is explicitly modeled as a stochastic process that disturbs light vehicles as per Newell's kinematic wave theory of moving bottlenecks.

The model is applied for estimating the impact of truck lane restrictions. It is found that system capacity is maximized either (i) when each truck type uses a different lane or (ii) when they share the same lane, depending on the relative proportion of heavy trucks. Moreover, this application sheds some light to the puzzling empirical result in several places around the world, where time and time again it is observed that the operational improvements of truck lane restrictions are small or negligible.

Key words:

Freeway capacity, truck special lanes, kinematic wave theory, moving bottlenecks

1 Introduction

Theories for predicting the impact of slow vehicles on traffic streams have flourished in recent years thanks to Newell's kinematic wave theory of moving bottlenecks (Newell, 1993, 1998), which describes the effects of a *single* slow moving convoy on a traffic stream. [1] Although it is intuitive that a slow vehicle will cause a queue of fast vehicles behind it, exactly what traffic state it is and how it relates to the trajectory of the moving bottleneck was not known until Newell (1993, 1998); see Leclercq et al. (2004) for a review and Muñoz and Daganzo (2002) for the experimental validation of the theory. Numerical solution methods for the moving bottlenecks model (Daganzo and Laval, 2005a,b) made possible simulation models able to capture the impacts of slow vehicles with remarkable accuracy (Laval, 2005; Laval et al., 2005; Laval and Daganzo, 2006). Analytical capacity formulae for bottlenecks caused by a single-type of slow vehicles have also been developed using moving bottlenecks theory for both multilane freeways (Laval, 2006b) and two-lane rural roads (Laval, 2006a).

One may argue that earlier theories for two-lane rural roads may be applicable here (see Daganzo, 1975, for a complete review on two-lane roads). Unfortunately, these theories either neglect physical queues or consider a single slow-vehicle type, and they are all concerned with homogeneous road segments.

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[1] From a historical perspective, Gazis and Herman (1992) were the first to propose a model to predict the impacts of a slow vehicle. They did so for a slow truck on a two-lane road, but their analysis does not show the connection with kinematic wave theory.

Also, earlier results have not been proven successful which explains why the state-of-the-art fully relies on microsimulation models.

This paper extends the framework introduced in Laval (2006b), which has two limitations that may introduce biases in certain applications. First, a single truck type is considered, which can be a problem when the truck fleet has very heterogeneous acceleration capabilities. Second, as in the standard kinematic wave model (Lighthill and Whitham, 1955; Richards, 1956), all lanes are modeled as a *single pipe*, which makes it difficult to include different truck types as their impact may vary depending on the lane they use. Here, we extend this framework by dropping the single-pipe assumption and analyzing each freeway lane separately, and by considering an arbitrary number of truck types. These extensions allow us to address more realistic problems and to analyze the effect of varying lane-specific truck proportions on overall capacity. Interestingly, we find that the lane-specific formulation improves the mathematical tractability of the problem so that additional closed-form expressions can be derived.

The model is applied for estimating the impact of freeway truck lane restrictions, and formulae for the resulting capacity improvement are derived. It is shown that an optimistic capacity improvement is around 4%, and that in most cases it is negligible. This is consistent with empirical results in several places around the world, where time and time again it is observed that the operational improvements are small or negligible. The reader is referred to Mannering et al. (1993), Sims and Royster (2006) and Fontaine (2008) for well-documented experiments in the US, and to Borchardt et al. (2004) for a comprehensive review of worldwide experiences. Although these results might seem surprising, this paper offers an explanation based on the finding that sys-

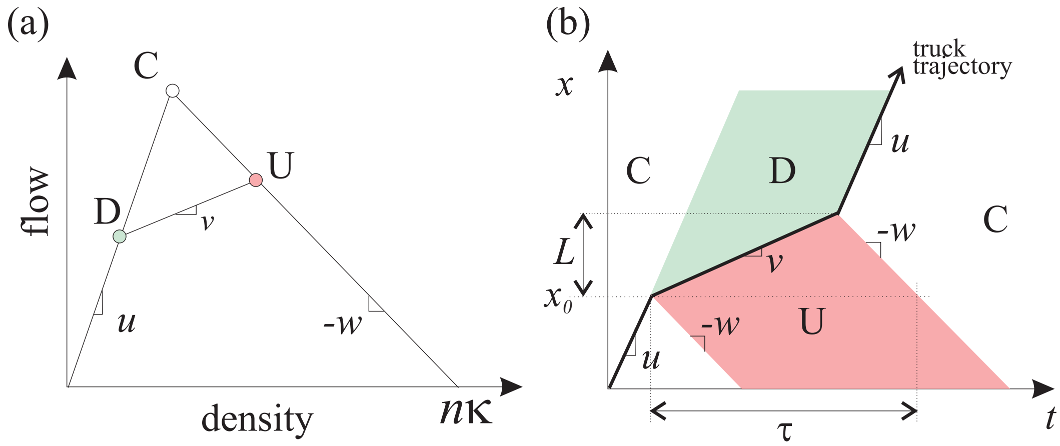

Fig. 1. (a) Representation of a moving bottleneck in a triangular fundamental diagram; (b) approximate truck trajectories and induced traffic states.

tem capacity is a convex function of the number of lanes available to trucks. Notice that in the literature no theories have been proposed to understand this problem; it has only been analyzed using microsimulation models that have not been shown to replicate the impacts of slow vehicles on traffic streams (see e.g. Cate and Urbanik, 2004; Gan and Jo, 2003; Rakha et al., 2005).

This paper is organized as follows. Section 2 presents a background on single-pipe capacity formulae considering one truck type. Section 3 examines single-lane facilities in the case of an arbitrary number of truck types for both continuous and discrete speed distributions. In section 4 the resulting model is extended to multilane facilities and it is applied to estimate the capacity improvement of truck lane restrictions. Finally, section 5 presents a discussion and future work.

2 Background: single pipe, single truck-type capacity formulae

Analytical formulae for the capacity of freeway bottlenecks caused by the presence of identical trucks have been developed in Laval (2006b) using renewal theory. It was assumed a road with n identical lanes obeying the triangular fundamental diagram defined in Fig. 1a with free-flow speed u, wave velocity w and jam density for one lane κ. Only one truck type was considered and a single-pipe approach was utilized, which amounts to assuming that all lanes behave identically and that trucks only use one lane. In this setting, the *normalized* capacity of the freeway section, ρ, is given by

$$\rho = \frac{1}{r\bar{H}C}, \tag{1}$$

where r is the time-mean proportion of trucks in the traffic stream, $C = uwn\kappa/(w+u)$ is the "ideal" capacity of all freeway lanes (i.e., with no trucks), and $\bar{H}$ gives the expected value of the random variable H, defined as the headway between two consecutive trucks at a given location $x = x_0$. In the sequel x_0 is the location where trucks are known to slow down (e.g., the beginning of an upgrade). Using the Poisson approximation to the binomial arrival process for trucks the probability density function (PDF) of H was shown to be

$$f_H(h) = \begin{cases} \lambda_1 \, \mathrm{e}^{-h\lambda_1} & , h \leq \tau, \\ \mathrm{e}^{\tau(\lambda_0-\lambda_1)} \, \lambda_0 \, \mathrm{e}^{-h\lambda_0} & , h > \tau, \end{cases} \tag{2}$$

which corresponds to the exponential distribution in each time interval $h \leq \tau$ and $h > \tau$. The quantity τ represents the *disturbance time* defined as the time it would take for the queue created by a single slow vehicle to clear at x_0. It was assumed that slow-vehicle trajectories are piecewise linear with slopes v

and u, as in Fig. 1b, and therefore

$$\tau(v) = L\frac{w+v}{wv}, \tag{3}$$

where L is the length of the segment where trucks are forced to slow down. [2] In (2) we used the notation

$$\lambda_0 = rC, \tag{4a}$$

$$\lambda_1 = rU, \tag{4b}$$

to denote the mean truck arrival rate at x_0 when the flow at this point is C or U, respectively. [3] Traffic state U corresponds to the queue upstream of an active moving bottleneck. State D in Fig. 1b is the flow downstream of the moving bottleneck when it holds back a queue is assumed to be equal to the capacity of the unblocked lanes; i.e., $D = (n-1)C/n$, which is in rough agreement with the observations in Muñoz and Daganzo (2002). According to the theory of moving bottlenecks:

$$U(v) = D + \frac{wv}{w+v}\kappa. \tag{5}$$

With all, the average truck headway is given by

$$\bar{H} = (1-\mathrm{e}^{-\lambda_1\tau})\frac{1}{\lambda_1} + \mathrm{e}^{-\lambda_1\tau}\frac{1}{\lambda_0}, \tag{6}$$

and capacity can be evaluated using (1). In Laval (2006b) extensions were explored by postulating different PDFs for τ in order to reflect more realistic

[2] If the slow segment also affect cars, the free-flow speed u should be taken as the speed of cars inside this segment. This does not introduce any bias given that we are interested in capacity.

[3] We use the convention that the flow corresponding to a traffic state A is denoted in italics, e.g. A.

situations but always constrained to a single truck type. As shown next, in this paper we explore a different alternative to generalizing the results in Laval (2006b). In particular, we recognize that still in cases with several types of trucks one can identify "embedded renewal processes" that can be solved sequentially in order to get analytical formulae for capacity.

3 Single-lane capacity

This section presents the formulation for the capacity of a single-lane facility with an arbitrary number of truck types. Towards this end, the problem with two truck types is analyzed in the next subsection, which is then generalized both in the discrete and the continuous case.

3.1 Two truck types

In this section we consider two types of trucks: heavy trucks that travel at a speed v_1 inside the segment of length L, and light trucks that travel at $v_2 > v_1$ inside this segment. Elsewhere both trucks travel at regular vehicles free-flow speed u. Let

- r be the proportion of trucks in the traffic stream,
- p_1 the proportion of trucks that are heavy,
- $p_2 = 1 - p_1$ the proportion of trucks that are light,
- $U_i = U(v_i)$ the queued flow generated by truck type i.
- $\lambda_i = rp_iU_i$ the flow of trucks type i when freeway flow is U_i.
- $\tau_i = \tau(v_i)$ the disturbance time generated by truck type i,
- $\tilde{\tau}_i = \tau_i/\tau(u)$ the dimensionless disturbance time generated by truck type i,

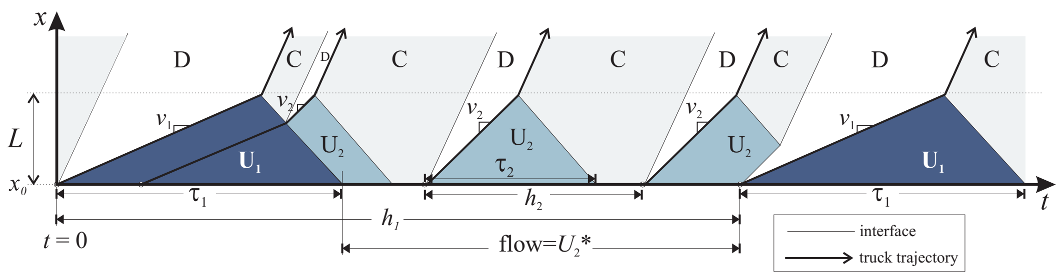

Fig. 2. Example of a realization in a single lane and two types of trucks.

Notice that $1/\tilde{\tau}_i = v_i(u+w)/(u(v_i+w))$ is also equal to U_i/C, the dimensionless flow in the queue of a truck type i. Here we assume a single-lane roadway and generalize the results in the next section.

Compared to the multi-pipe approach, the mathematical tractability of the problem improves considerably for a single lane. To see this, we eliminate v in (5) and (3), which gives $U(v) = \kappa L/\tau(v)$ since $D = 0$ for a single lane. It turns out that the quantity

$$\varphi = r\kappa L, \tag{7}$$

is a key (dimensionless) variable that gives the expected number of trucks within the disturbance time of any type. In the case of a single truck type, say $i = 2$ and $p_2 = 1$, eqn. (1) simplifies to

$$1/\rho = \mathrm{e}^{-\varphi} + (1 - \mathrm{e}^{-\varphi})\tilde{\tau}_2, \tag{8}$$

after noting that in (6) $\lambda_1\tau = \varphi$ and $\tilde{\tau}_2 = C/U_2$. Eqn. (8) is a function of only two variables, φ and $\tilde{\tau}_2$. It become straight forward to check, for example, that when the uphill is very long and/or the proportion of trucks is very high (i.e., $\varphi \to \infty$) then the capacity tends to the flow in the queue of the truck, $1/\tilde{\tau}_2$, as expected.

To understand the possible interaction between light and heavy trucks, Fig. 2 shows an example of a realization in a single lane in between two successive heavy truck arrivals. Notice how the 2nd truck arrives at x_0 queuing behind a heavy truck and therefore it is forced to travel at v_1 as long as it is queuing behind the heavy truck. Once it has left the heavy truck queue, it travels the remainder of the uphill at v_2, which generates the congested traffic state U_2 in the figure. Also note that the second heavy truck arrives within the queue of a light truck, but is not affected by it because $v_1 < v_2$.

The key observation here is that heavy truck arrivals still obey a renewal process. This is true because the speed of heavy trucks on the uphill is always v_1, even if a heavy truck arrives at x_0 within the disturbance time of a previous truck. Therefore, the flow at x_0 after the arrival of a heavy truck follows the same distribution regardless of the traffic state prevailing at the time of arrival. It follows that freeway capacity can be evaluated by examining the flow in between two successive heavy truck arrivals. As can be seen in Fig. 2, the flow at (t, x_0)–measured from the last heavy truck arrival–is U_1 if $t \leq \tau_1$ and U_2^* if $t \geq \tau_1$, where U_2^* is the average total flow in $\tau_1 \leq t \leq h_1$. To obtain U_2^* we note that in $\tau_1 \leq t \leq h_1$ only light trucks can arrive (otherwise a renewal would take place), and therefore U_2^*/C corresponds to the one-truck-type capacity (8); i.e.,

$$U_2^* = 1/(rp_2\bar{H}_2), \tag{9a}$$

$$= C/(\mathrm{e}^{-p_2\varphi} + (1 - \mathrm{e}^{-p_2\varphi})\tilde{\tau}_2), \tag{9b}$$

By letting $\lambda_1^* = rp_1U_2^*$ represent the mean heavy truck arrival flow for $t > \tau_1$, we have that heavy truck headways at the bottom of the uphill may be

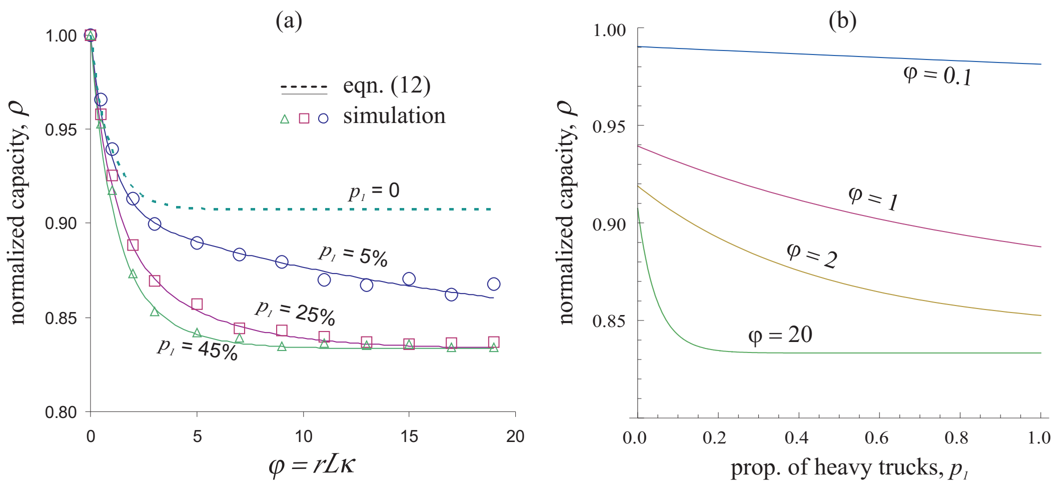

Fig. 3. (a) Agreement between simulation and the approximate formula (12). Parameter values: $u = 120$ km/hr, $\kappa = 150$ veh/km/lane, $w = 20$ km/hr, $v_1 = 50$ km/hr, $v_2 = 70$ km/hr. For simulation $L = 1$ km and r was varied such that $\varphi = 1 \ldots 20$. (b) Capacity as a function of the proportion of heavy trucks for different values of φ.

expressed similarly as in (2); i.e.:

$$f_{H_1}(h_1) = \begin{cases} \lambda_1 \, \mathrm{e}^{-h_1 \lambda_1} & , h_1 \leq \tau_1, \\ \\ \mathrm{e}^{\tau_1(\lambda_1^* - \lambda_1)} \, \lambda_1^* \, \mathrm{e}^{-h_1 \lambda_1^*} & , h_1 > \tau_1, \end{cases} \tag{10}$$

Hence,

$$\bar{H}_1 = (1 - \mathrm{e}^{-\lambda_1 \tau_1}) \frac{1}{\lambda_1} + \mathrm{e}^{-\lambda_1 \tau_1} \, \frac{1}{\lambda_1^*}. \tag{11}$$

and the capacity $\rho = 1/(r p_1 \bar{H}_1 C)$ simplifies to

$$1/\rho = \mathrm{e}^{-\varphi} + (1 - \mathrm{e}^{-p_1 \varphi}) \tilde{\tau}_1 + (\mathrm{e}^{-p_1 \varphi} - \mathrm{e}^{-\varphi}) \tilde{\tau}_2. \tag{12}$$

Notice that (12) matches (8) in the following three limit cases: (i) $\tilde{\tau}_1 = \tilde{\tau}_2$, (ii) $p_1 = 0$ and (iii) $p_1 = 1$. Also, as $\varphi \to \infty$ then $\rho \to 1/\tilde{\tau}_1$, as expected.

It is worth noting that in the interval $0 \leq t \leq \tau_1$ one does not need to worry about truck type-2 arrivals. This is true because fast trucks are forced to travel at v_1 in the uphill and therefore the flow in this time period remains U_1.

To assess the quality of this approximation we have used the simulation model in Laval and Leclercq (2008), which in the case of a single lane gives the exact solution of the kinematic wave model, free of numerical errors (Daganzo, 2006). Since this model is microscopic, one is able to simulate truck arrivals as a binomial process, exactly as defined in the beginning of this section. As shown in Fig. 3a, the agreement between simulation and the approximate formula (12) is uncanny. This excellent agreement comes to a surprise because the flow in $\tau_1 \leq t \leq h_1$ is time-dependent in reality but has been approximated by the average U_2^* in the derivation of (12).

It is interesting to note in Fig. 3a that for $p_1 = 5\%$ and $\varphi > 4$ capacity decays almost linearly to the minimum capacity $1/\tilde{\tau}_1$. It turns out that this linear rate of decay increases with p_1; in the limit $p_1 = 0$ capacity tends to $1/\tilde{\tau}_2$, as expected; see the dashed line in the figure. Part b of the figure shows the capacity as a function of the proportion of heavy trucks for different values of φ. Notice that at $p_1 = 0$ there are only light trucks and thus the intercept of these curves is given by (8) and the corresponding value of φ.

3.2 Multiple truck types

In this section we report capacity formulae in the case of arbitrary truck speed distributions, either discrete or continuous. As shown next, the continuous case will be obtained as the limit of the discrete case when the number of truck types tends to infinity.

3.2.1 Discrete speed distribution

Here we assume that truck desired speeds on the uphill section are restricted to a set of n discrete values v_i with cumulative distribution function (CDF) $G(v_i) = \sum_{j=1}^{i} p_j, i = 1 \dots n$, where p_i gives the proportion of trucks with desired speed v_i. As before, we adopt the convention that a higher i-index implies a higher speed.

As in the case of two types of trucks, a renewal takes place each time the slowest truck (type 1) arrives at the bottom of the uphill. Unlike that case, however, in the time interval $\tau_1 \leq t \leq h_1$ (see Fig. 2) one has to solve a problem with $n-1$ types of trucks, since a type-1 arrival can not take place (otherwise a new renewal takes place and the process starts over). One can repeat this recursion until reaching a one-type problem whith only the fastest truck type, whose solution is given by eqn. (8) with $i = n$. Then, one can unfold the recursion by solving a series of two-type problems, whose solution is analogous to (12). This can be accomplished by solving the following recursion for $i = n, n-1, \dots 1$:

$$\bar{H}_i = (1 - \mathrm{e}^{-\lambda_i \tau_i})/\lambda_i + \mathrm{e}^{-\lambda_i \tau_i}/\lambda_i^*, \tag{13a}$$

$$\lambda_i = r p_i U_i, \tag{13b}$$

$$\lambda_i^* = r p_i U_{i+1}^* \tag{13c}$$

$$U_i^* = 1/(r p_i \bar{H}_i), \tag{13d}$$

with $U_{n+1}^* = C$. Noting that $\lambda_i^* = p_i/(p_{i+1}\bar{H}_{i+1})$ and $\lambda_i \tau_i = p_i \varphi$ the above system of equations can be reduced to the recursion

$$\bar{H}_i = a_i + b_i \bar{H}_{i+1}, \tag{14}$$

where $a_i = (1 - \mathrm{e}^{-p_i \varphi})/\lambda_i$ and $b_i = \mathrm{e}^{-p_i \varphi}\, p_{i+1}/p_i$. It is straightforward to show that $\bar{H}_1 = \sum_{i=1}^{n} a_i \prod_{j=1}^{n-1} b_j$, which implies that the capacity $\rho = 1/(r p_1 \bar{H}_1 C)$ is given by

$$1/\rho = \mathrm{e}^{-\varphi} - \sum_{i=1}^{n} \tilde{\tau}_i \left(\mathrm{e}^{-\varphi G(v_i)} - \mathrm{e}^{-\varphi G(v_{i-1})} \right) \tag{15}$$

with $G(v_0) = 0$. Notice that as $\varphi \to \infty$ every term in the RHS of (15) tends to zero except for $\mathrm{e}^{-\varphi G(v_{i-1})}$ when $i = 1$, which equals unity. It follows that the capacity tends to $1/\tilde{\tau}_1$ when $\varphi \to \infty$, as expected.

Again, the agreement between the approximate capacity formula proposed here and simulation of the exact process is strikingly good. This is shown in Fig. 4a for $n = 3$, 4 and the corresponding discrete uniform distribution between 50 and 90 km/hr. It can be seen that adding an extra type of truck changes the capacity curve only marginally. Next we examine the case $n \to \infty$.

3.2.2 Continuous speed distribution

In this section a truck's desired speed on the uphill section is a continuous random variable, v, in the domain $v_{\min} \le v \le v_{\max}$ with CDF $F(v) = \int_0^v f(s)\,\mathrm{d}s$. The density $f(v)$ is such that $f(v)\,\mathrm{d}v$ is the proportion of trucks with desired speed in $(v, v + \mathrm{d}v)$. Naturally, the disturbance time also become a continuous random variable; i.e., $\tilde{\tau}(v) = u(v + w)/(v(u + w))$.

As $n \to \infty$ the term $\mathrm{e}^{-\varphi G(v_i)} - \mathrm{e}^{-\varphi G(v_{i-1})}$ in (15) can be interpreted as the differential of the function $\omega(v) = \mathrm{e}^{-\varphi F(v)}$; i.e., $\mathrm{d}\omega(v) = \varphi f(v)\,\mathrm{e}^{-\varphi F(v)}\,\mathrm{d}v$.

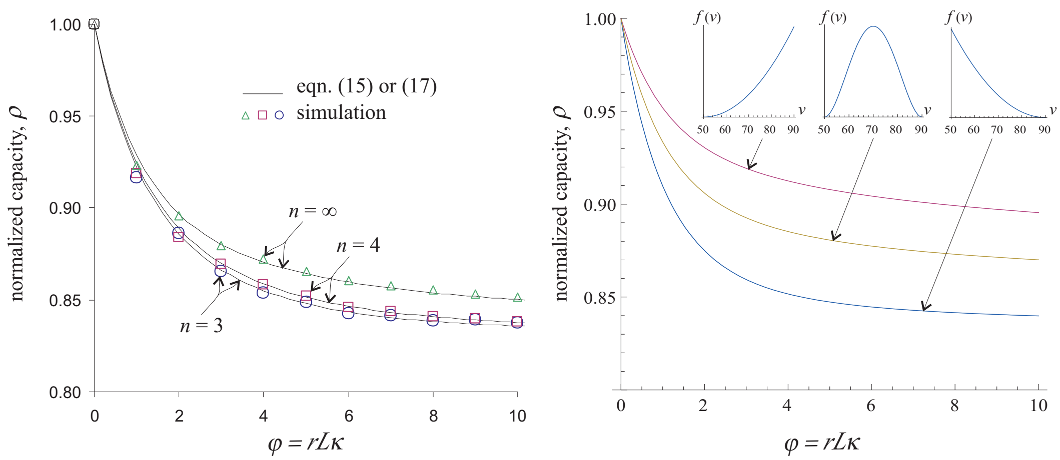

Fig. 4. (a) Agreement between simulation of the exact process and the discrete and continuous capacity formulae (15) and (17) for the uniform distribution between 50 and 90 km/hr. (b) Effects of the shape of $f(v)$: from left to right, Beta distribution with parameters (3,1), (3,3), and (1,3).

Therefore, the sumation term in (15) tends to $\int_{v_{\min}}^{v_{\max}} \tilde{\tau}(v)\,\mathrm{d}\omega(v)$, which is the expected value of $\varphi\tilde{\tau}(v)\omega(v)$. It follows that

$$1/\rho = \mathrm{e}^{-\varphi} + \varphi \mathrm{E}[\tilde{\tau}(v)\,\mathrm{e}^{-\varphi F(v)}], \tag{16}$$

where $\mathrm{E}[\cdot]$ represents the expected value operator. Reassuringly, it can be shown that $\rho \to 1/\tilde{\tau}(v_{\min})$ as $\varphi \to \infty$, as expected. Unfortunately, (16) does not simplify to a fully algebraic expression even for the simplest PDF. For example, the uniform distribution in $v_{\min} \leq v \leq v_{\max}$ leads to

$$1/\rho = \mathrm{e}^{-\varphi} + \frac{u}{u+w}\left(1 - \mathrm{e}^{-\varphi} - \frac{\mathrm{e}^{\theta\varphi}\, w\varphi(\Xi(-\theta\varphi) + \Xi((1-\theta)\varphi))}{v_{\max} - v_{\min}}\right), \tag{17}$$

where $\theta = v_{\min}/(v_{\max} - v_{\min})$ and $\Xi(z) = -\int_{-z}^{\infty} \mathrm{e}^{-s}/s\,\mathrm{d}s$ is the exponential integral function, which can only be evaluated numerically.

The top curve on Fig. 4a shows the excellent agreement between (17) and the simulation of the exact process for the uniform distribution with $v_{\min} = 50$ and $v_{\max} = 90$ km/hr. It becomes apparent that, at least for the uniform distribution, the marginal change and the capacity curve when adding an extra truck type is a very small. To see the effects of the shape of $f(v)$, part b of the figure shows (16) for the three PDFs included in the figure, which cover typical cases that may be found in reality. As expected, it can be seen that the distribution with higher proportion of slow vehicles gives a lower capacity for a fixed φ, and that the shape of $f(v)$ influences capacity significantly.

4 Multilane capacity

In this section we extend the previous results for the case of multilane freeways. For simplicity, we restrict our analysis to two-lane facilities and two types of trucks since it is straightforward to extend the following results to the multilane/multi-type case.

Lane assignments are assumed as follows. Heavy trucks only use lane 1, which is consistent with "slow vehicles keep right" signs often placed near freeway upgrades. This behavior may also arise spontaneously since heavy trucks should not have a lane preference as they travel at the same speed v_1 regardless of the lane they choose. In the case of light trucks, it is assumed that there is a proportion, ξ, that uses lane 1, while the remaining $1 - \xi$ uses lane 2. Although it is clear that light trucks have no incentive to use lane 1 (they can always travel at their desired speed in lane 2) the introduction of the variable ξ reveals considerable insight and allows us to study truck lane restriction, for example. Finally, it is worth recalling that in order for the moving bottleneck theory to hold cars are allowed in both lanes.

The last assumption is that the number of lane changes along the uphill is small, so that total capacity can be decomposed into independent single-lane problems as in the previous section. A discussion of the appropriateness of this assumption is provided in §5.

In the following definitions indices $\ell = 1, 2$ and $i = 1, 2$ stand for the lane number and the truck type, respectively. Let

- r be the total proportion of trucks in the traffic stream,
- p_1 the total proportion of trucks that are heavy and $p_2 = 1 - p_1$ the total proportion of trucks that are light,
- r_ℓ be the proportion of trucks in lane ℓ,
- $p_{\ell,i}$ the proportion of trucks in lane ℓ that are type i,
- $\varphi_\ell = r_\ell \kappa L$, and
- ρ_ℓ the capacity in lane ℓ.

From the above definitions it follows that lane-specific data are given by

$$r_1 = (p_1 + \xi p_2) r, \tag{18a}$$

$$r_2 = (1 - \xi) p_2 r, \tag{18b}$$

$$p_{1,1} = p_1 / (p_1 + \xi p_2), \tag{18c}$$

$$p_{1,2} = \xi p_2 / (p_1 + \xi p_2), \tag{18d}$$

$$p_{2,1} = 0, \tag{18e}$$

$$p_{2,2} = 1. \tag{18f}$$

Using the results in section 2, the capacity in lane 1 is given by (12) and in lane 2 by (8). Of course, when evaluating these equations the terms φ and p_1 in (12) must be replaced by φ_1 and $p_{1,1}$ as defined in this section, and the term φ and $\tilde{\tau}_i$ in (8) by φ_2 and $\tilde{\tau}_2$. As a function of ξ these capacities turn out to be given by:

$$1/\rho_1(\xi) = \mathrm{e}^{-(p-q\xi)\varphi} + \left(1 - \mathrm{e}^{-p\varphi}\right)\tilde{\tau}_1 + \left(\mathrm{e}^{-p\varphi} - \mathrm{e}^{-(p-q\xi)\varphi}\right)\tilde{\tau}_2, \tag{19a}$$

$$1/\rho_2(\xi) = \mathrm{e}^{-q\varphi(1-\xi)} + \left(1 - \mathrm{e}^{-q\varphi(1-\xi)}\right)\tilde{\tau}_2. \tag{19b}$$

We now explore the effect of ξ on total capacity.

Theorem 1: The total normalized capacity $\rho_T(\xi) = \frac{1}{2}(\rho_1(\xi) + \rho_2(\xi))$ is a convex function of ξ.

Proof. To show that this proposition is true we will show that the capacity on each lane is a convex function of ξ. To simplify the exposition, will show that the inverse capacity functions $g_\ell(\xi) \equiv 1/\rho_\ell(\xi)$ are *concave* functions of ξ. It is straightforward to show that

$$\frac{\mathrm{d}^2 g_1(\xi)}{\mathrm{d}\xi^2} = -\,\mathrm{e}^{-(p_1+p_2\xi)\varphi}\, p_2^2(\tilde{\tau}_2 - 1)\varphi^2, \tag{20a}$$

$$\frac{\mathrm{d}^2 g_2(\xi)}{\mathrm{d}\xi^2} = -\,\mathrm{e}^{p_2(\xi-1)\varphi}\, p_2^2(\tilde{\tau}_2 - 1)\varphi^2, \tag{20b}$$

which are clearly less than or equal than zero since $\tilde{\tau}_2 > 1$. It follows that $\rho_1(\xi)$ and $\rho_2(\xi)$ are convex functions of ξ, and so is the total capacity $\rho_T(\xi)$. $\diamondsuit$

Theorem 1 indicates that the total capacity will be maximized either at $\xi = 0$ or at $\xi = 1$. This is illustrated in Fig. 5, which depicts the capacity in lane 1 and 2 as well as the total normalized capacity (thick line) for typical parameter values. It can be seen that for a small heavy truck proportion (first column on the figure) maximum total capacity tends to occur at $\xi = 0$, and as this proportion increases maxima tends to occur at $\xi = 1$.

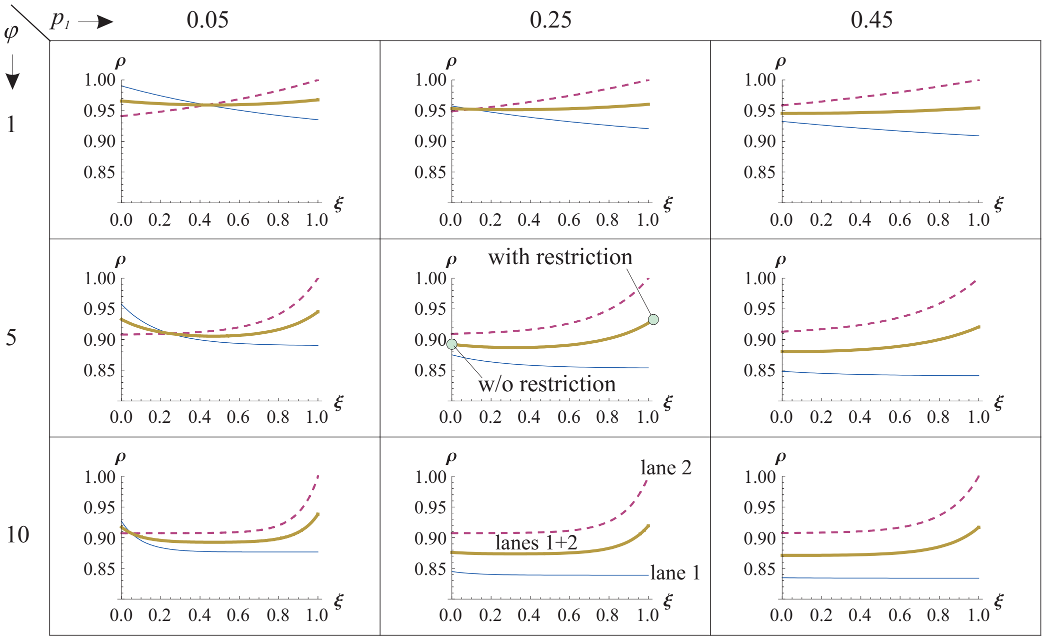

Fig. 5. Normalized capacity in lane 1, lane 2 and both lanes as a function of ξ. Parameter values as in Fig. 3.

4.1 Truck lane restrictions

Truck lane restrictions can be readily assessed with the preceding framework. Since heavy trucks are assumed to travel in the right lane 1 only, and light trucks are better off traveling in lane 2, the restriction and no-restriction scenarios correspond to $\xi = 1$ and $\xi = 0$ in our model.

The changes in capacity due to truck lane restrictions can be seen in Fig. 5. This figure shows that the change in total capacity is negligible when the proportion of heavy trucks is small but increases as this proportion grows. However, the maximum changes in capacity remain small or negligible for typical parameter values. To see this, Fig. 6a shows the relative capacity increase, $\Delta\rho = (\rho_T(0) - \rho_T(1))/\rho_T(1)$, as a function of φ and the proportion of heavy trucks p_1 given $v_1 = 50$ and $v_2 = 70$ km/hr. This figure shows that $\Delta\rho$

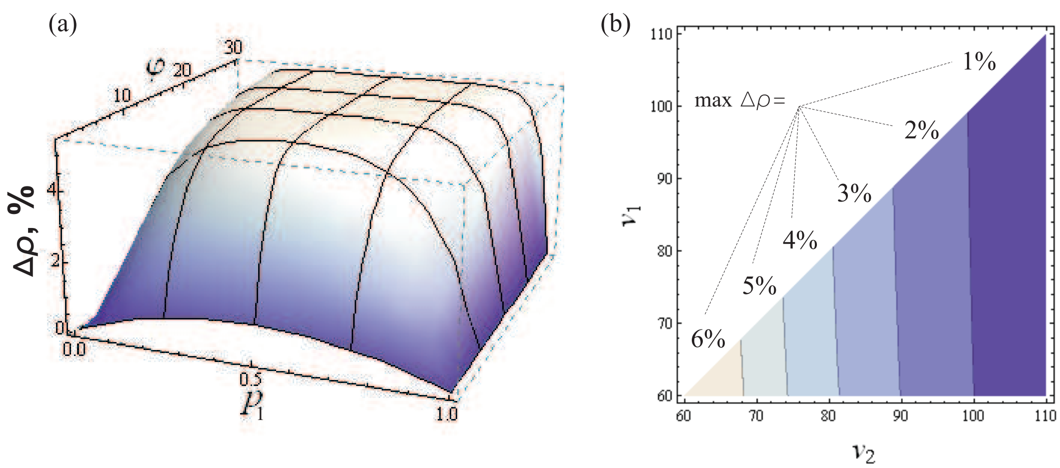

Fig. 6. (a) Relative capacity increase as a function of φ and proportion of heavy trucks for $v_1 = 50$ and $v_2 = 70$ km/hr; (b) contour map of $\max \Delta\rho$.

is a plateau with maxima at $\varphi \to \infty$ and $p \approx 0.5$. Notice that this conclusion remains valid regardless of the values of v_1 and v_2. It turns out that the maximum relative capacity increase is given by:

$$\max \Delta\rho = \frac{\tilde{\tau}_2 - 1}{1 + \tilde{\tau}_2/\tilde{\tau}_1}, \tag{21}$$

which is only a function of $\tilde{\tau}_1$ and $\tilde{\tau}_2$, and therefore of v_1 and v_2 alone. Figure 6b shows a contour map of $\max \Delta\rho$, where it can be seen that it increases as truck speeds decrease. As noted in the introduction, empirical evidence around the world reports maximum operational improvements of around 3-4%, which would correspond to truck speeds of around 70-90 km/hr according to the figure.

Finally, multilane freeways can be treated with the model presented here provided that the proportion of trucks choosing each lane are known for all types of trucks. In this case, equations analogous to (18) can be set up and the capacity of each lane could be readily obtained. The main result of this section should still apply; i.e., (i) changes in capacity due to truck lane restrictions are small or negligible, and (ii) system capacity is maximized either if all trucks share the same lane or if each truck type uses a different lane. In this latter case, if the number of truck types is greater than the number of lanes, our results suggest that maximum capacity could be achieved by aggregating trucks in as many classes as there are lanes and assigning each class to a different lane.

5 Discussion and Outlook

This paper derived formulae for the capacity of road segments in the presence of slow vehicles, when the *desired* speed distribution (discrete or continuous) is known and is time invariant. Throughout the text we used trucks to exemplify slower vehicles, but slower cars can be treated in the same way. Similarly, the uphill bottleneck was used as an example throughout the text, but the method is applicable to any situation where the speed distribution inside a slow segment is known; e.g., steeped downgrade and sharp horizontal curve bottlenecks. If the proportion of trucks is high enough so that they tend to arrive in batches of, say b trucks at a time, a good approximation would be to use an effective truck proportion of r/b. The main assumption here is, as before, that the length of a truck is negligible compared to the length of the freeway segment under analysis. Pending further research, it may be possible to extend the framework presented here to segments in the vicinity of a freeway entrance/exit and weaving sections.

It may be challenging to estimate the desired speed distribution from field data because one can only observe actual speeds. As an alternative, one can use the distribution of the crawl speed for heavy vehicles, which only depends on the fleet's engine power to weight ratio and the grade of the road section.

The empirical validation of the model relied upon the few truck-lane restriction experiments documented in the literature. Unfortunately, the level of aggregation in the reported field data only allowed us to validate the model at a coarse scale, which prompted us to use simulation for a more detailed validation. In the case of a single lane, we are confident that this validation is accurate because the kinematic wave model in a single lane has been extensively validated in the field. In the case of multilane roads, there is no simulation model validated in the context of the problem presented here. In situations where the number of lane changes is small, however, we expect the sum of individual lane capacities to be accurate. Notice that for the problem examined here the number of lane changes should be small in most situations; e.g., when the speed difference among trucks is not too important, or when all types of trucks use all the lanes.

In cases where the number of lane changes is important, the problem is far more complex and analytical formulae may not be possible to obtain. In fact, a lane changing can have a positive or negative effect on capacity depending on the prevailing traffic state in the target lane. When the target lane is empty (e.g., white regions in Fig. 2) every lane changing increases capacity; when the target lane is congested (e.g., shaded regions in Fig. 2) they may create momentary moving bottlenecks that decrease capacity, as per the theory in Laval and Daganzo (2006). To approximate these phenomena effectively, one would need a model for driver lane-changing behavior in the presence of trucks, i.e. a model that would predict the lane-changing rate as a function of the size of the white and shaded areas in Fig. 2. Only then one could express the lane-changing rates as a function of relevant parameters and then calibrate this model with simulation or field data. This is currently been investigated by the author.

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