Stochastic Processes of Moving Bottlenecks

Approximate Formulas for Highway Capacity

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This paper presents a general framework for developing formulas to estimate the capacity of freeway bottlenecks caused by the presence of slow vehicles. These underperforming vehicles may represent trucks on an uphill grade, cautious lane changers at a weaving section, or any other stream of vehicles moving consistently—at least for a short period of time—more slowly than prevailing traffic. From modeling of the underperforming stream as simplified yet realistic stochastic processes, closed-form expressions for the segment's capacity can be obtained. Four such processes are presented and exemplified for the case of short uphill grades, a type of facility where existing capacity formulas fail. The proposed models—which do not need calibration—improve current estimates, and one in particular explains existing data with remarkable accuracy.

This paper presents a general framework for evaluating the capacity of freeway segments that contain bottlenecks caused by the presence of slow vehicles (SVs). The SVs may represent trucks on an uphill grade, cautious lane changers at a weaving section, vehicles on an acceleration lane, vehicles exiting the freeway while slowing down, or any other stream of vehicles moving consistently (at least for a short period of time) more slowly than prevailing traffic. SVs are assumed to be a constant proportion of the traffic stream and to follow simple two-speed trajectories. These assumptions allow closed-form expressions for the capacity of the segments to be obtained by means of simplified yet realistic stochastic processes.

The approach proposed in this paper is based on Newell's kinematic wave theory of moving bottlenecks (KW-MB theory) (*1, 2*), which describes the effects of a single slow-moving convoy on a traffic stream. According to this theory, an active moving bottleneck generates a queue upstream and freely flowing traffic downstream, exemplified by points U and D in Figure 1*a* for the case of a triangular fundamental diagram (FD). [Throughout this paper triangular FDs are used since they are the simplest relations that agree with empirical evidence (*3–5*).] The slope of the line connecting these points is the speed of the moving bottleneck, v. The flow of cars in the queue is Q_U, and their speed is v_U. The flow downstream of the truck when it holds back a queue is called the bottleneck capacity (*6*). It is the flow of state D, Q_D, in the figure.

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For a road with n identical lanes obeying a triangular FD with free-flow speed u, wave velocity w, and jam density for one lane κ,

$$Q_D = \frac{n-1}{n}Q \qquad Q_U = Q_D + \frac{wv}{w+v}\kappa \tag{1}$$

where $Q = n\dfrac{wu}{w+u}\kappa$ is the capacity of the freeway segment with no SVs.

The solution of the KW-MB model with a triangular FD is simple. Figure 1*b* shows the solution on the time–space plane of a problem with a moving bottleneck that enters a freeway flowing at capacity and travels from location $x = x_0$ to $x_0 + L$ at a constant speed v. The time it takes for the queue to clear at x_0 will be called the disturbance time, T, and will play a central role in this paper.

On the basis of this simple yet robust theory, several simulation-based approaches have been developed to tackle streams of SVs. In particular, the multilane hybrid model (*7*) treated lane-changing vehicles as moving bottlenecks and was able to explain to the capacity drop phenomenon on freeways. Similarly, the single-pipe hybrid (SPH) model (*8*) considers trucks on an uphill segment as moving bottlenecks and unveiled significant insights about the physical problem. The latter work (*8*) also developed a formula for the uphill capacity, which was accurate for long uphill segments but deviated from simulation results for short ones.

This paper attempts to describe the long-term effects of a stream of moving bottlenecks in a general context, where the formula of Laval and Daganzo (*8*) is a special case. Toward this end, a conditional renewal process to describe the stochastic arrivals of SVs is identified; that is, it is shown that, conditional on the disturbance time, the arrival of the next SV can be approximated by an inhomogeneous Poisson process. Four distributions for the disturbance times that may be useful in different contexts are then proposed. The distributions allow the formulation of the unconditional process and give rise to four models for the computation of capacity. The performance of each model for the case of short uphill segments is then described and compared with the simulation results of Laval and Daganzo (*8*).

GENERAL FRAMEWORK

Let x_0 be a location of the highway where SVs are known to appear (e.g., the bottom of an uphill grade, the gore of a weaving section, a freeway exit or entrance). Let r be the proportion of SVs in the

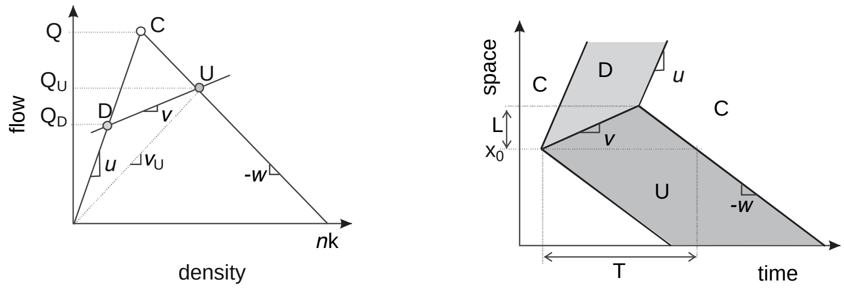

FIGURE 1 (*a*) Representation of a moving bottleneck in a triangular FD and (*b*) solution of a simple example in the time–space plane.

traffic stream and $N(t)$ be the cumulative number of SVs at x_0. Let the headway (or interarrival time) between two consecutive SVs at x_0 be the random variable H with unknown distribution $F(h) = P(H < h)$ to be determined. Let T be the random variable representing the disturbance time created by an SV, defined as the time it takes for the queue generated by the SV to clear at location x_0 (see Figure 2). Subscripts are used to represent SV-specific random variables in the order of arrival at x_0; for example, the sequence $\{H_1, H_2, H_3\}$ gives the headways of the first three consecutive SVs.

Conditional Process

Each time a vehicle passes x_0 it has a probability r of being an SV. Therefore, the arrival process for SVs is a binomial process with parameter r. If in addition r is small and the total number of vehicles is large, the process is well approximated by a Poisson process. However, the mean of this process is difficult to obtain; flows change continuously because of the queues generated by the arrival of each SV.

To simplify the problem, SV trajectories are assumed to be piecewise linear with slopes v and u, as in Figure 3. If the length of the slow portion of the trajectory is L, disturbances are given by

$$T = L\frac{w+v}{wv} \tag{2}$$

and the corresponding queued flow is Q_U from Equation 1. Notice that the latter assumption implies that all the SVs drive in the same lane, with no passing. However, this is not a limitation because the method can still be applied to obtain tight bounds in the cases with passing, as discussed elsewhere (*8*). The advantage of this crude approximation is that the SV arrival process at x_0 becomes a renewal process. It is clear from Figure 3 that flows at x_0 are either Q or Q_U. It follows from this figure that, conditional on T (or, equivalently, conditional on L and v), the mean rate of the process, $m_T(h)$, is

$$m_T(h) = \begin{cases} \lambda(T)h & \text{if } h \le T \\ \lambda(T)T + (h-T)\mu & \text{if } h > T \end{cases} \tag{3}$$

The following definitions were used in Equation 3:

$$\begin{aligned} \lambda(T) &= rQ_U(T) \\ \mu &= rQ \end{aligned} \tag{4}$$

Notice that $Q_U(T)$ can be obtained by eliminating v in the system of Equations 1 and 2, which gives $Q_U(T) = Q_D + \kappa L/T$. It also follows that the conditional headway distribution is

$$F(h|T) = 1 - P\left[N(t+h) - N(t) = 0|T\right] = 1 - e^{-m_T(h)} \tag{5}$$

Freeway Capacity

To estimate the capacity of the freeway segment all one needs to know is the average headway of SVs, $E(H)$. This is true because the

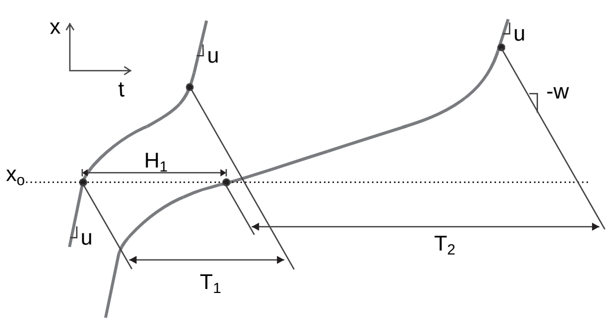

FIGURE 2 Example of two realistic SV trajectories.

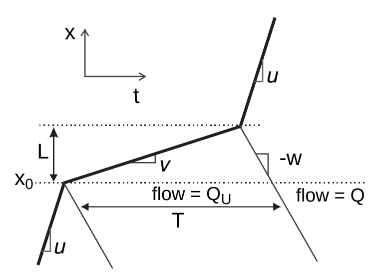

FIGURE 3 Definition of approximate two-speed trajectories.

average number of vehicles between two consecutive SVs plus the leading SV is $1/r$ (the number of cars between two consecutive SVs follows a geometric distribution with parameter r) and therefore the capacity is $1/[rE(H)]$. The normalized capacity of the upgrade, ρ, is given by

$$\rho = \frac{1}{rE(H)Q} \tag{6}$$

To identify $E(H)$, it is noted that

$$E(H|T) = \int_0^\infty [1 - F(h|T)]dh = \int_0^\infty e^{-m_T(h)}dh \tag{7}$$

which in combination with Equation 3 gives

$$E(H|T) = \frac{1}{\lambda(T)} + \left[\frac{1}{\mu} - \frac{1}{\lambda(T)}\right]e^{-\lambda(T)T} \tag{8}$$

If one knows the distribution of T it is straightforward to obtain the mean headway by taking expectation on Equation 8; that is,

$$E(H) = E[E(H|T)] \tag{9}$$

Next, different distributions for T will be explored.

APPROXIMATE DISTRIBUTIONS FOR DISTURBANCE TIMES

In this section different distributions for T with increasing levels of complexity are explored. These distributions are kept as simple as possible to allow for closed-form expressions of $E(H)$ while being realistic.

Model M1: Single-Valued Disturbances

If one assumes that all disturbances are identical and equal to τ_1, one can readily compute capacity (Equation 6) by using Equations 8 and 4. This special case was presented by Laval and Daganzo (*8*) for trucks on an uphill grade:

$$\rho = \frac{\rho_{\min}}{1 - e^{-rQ_U\tau_1}(1 - \rho_{\min})} \tag{10}$$

where $\rho_{\min} = Q_U/Q$ and τ_1 was the maximum possible disturbance (i.e., when the truck travels the entire uphill at the crawl speed). Recall that the crawl speed, v_C, is the steady-state speed of a vehicle on an infinitely long upgrade, where the speed drops to a point where the engine lacks power to accelerate. For a particular truck type, v_c is only a function of the grade of the uphill, G. It is clear that Equation 10 is not a good approximation for short uphills because the likelihood that a truck will attain v_c is significant only when the truck reaches the bottom of the uphill, x_0, queueing behind another truck; otherwise, when the truck reaches x_0 at free-flow speed it will travel at $v < v_c$ provided that the uphill is short. However, Equation 10 was found to match simulation results accurately for long uphill segments.

Model M2: Bivalued Disturbances

In a first attempt to capture the possibility of multiple disturbance times, it is assumed that T takes on only two values, depending on whether the SV arrival at x_0 occurs within a queue:

$$T_i = \begin{cases} \tau_0 & \text{if } H_{i-1} \geq T_{i-1} \\ \tau_1 & \text{if } H_{i-1} < T_{i-1} \end{cases} \tag{11}$$

where $\tau_0 < \tau_1$; see Figure 4. In this way, the correlation between the SV's initial speed and its resulting disturbance time is captured. For example, for bottlenecks caused by the lack of vehicle acceleration capabilities, the lower the initial speed, the longer it takes to accelerate to free-flow speed. Therefore, the probability of SV i to create a small disturbance τ_0 is given by the probability of arriving at x_0 after the disturbance of the previous SV has passed; similarly, if it arrives within the disturbance of the previous SV (i.e., queuing at a low speed), vehicle i would then create a bigger disturbance τ_1:

$$\begin{aligned} P(T_i = \tau_0) &= P(H_{i-1} > T_{i-1}) = 1 - F(T_{i-1}) \\ P(T_i = \tau_1) &= P(H_{i-1} \leq T_{i-1}) = F(T_{i-1}) \end{aligned} \tag{12}$$

It follows that the distribution of T_i is given by $F(T_{i-1})$. To obtain F, note that

$$\begin{aligned} F(T_i) &= E[F(T_i|T_i)] \\ &= F(T_i|T_i = \tau_0)P(T_i = \tau_0) + F(T_i|T_i = \tau_1)P(T_i = \tau_1) \\ &= F(\tau_0)[1 - F(T_{i-1})] + F(\tau_1)F(T_{i-1}) \end{aligned} \tag{13}$$

and impose the steady-state condition $F(T_i) = F(T_{i-1}) = F(T)$ and solve for $F(T)$. This steady-state condition means that the probability of an SV arriving within a queue is independent of the vehicle number, which is natural. Thus,

$$F(T) = \frac{F(\tau_0)}{1 + F(\tau_0) - F(\tau_1)} \tag{14}$$

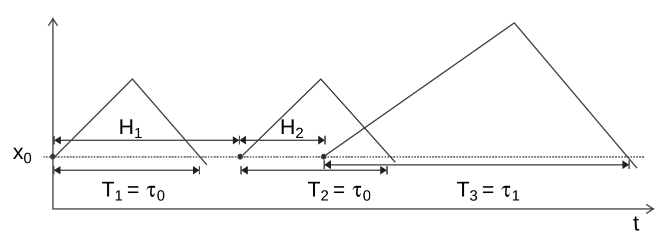

FIGURE 4 Example of a possible disturbance sequence for Model M2.

By defining $\lambda_j = \lambda(\tau_j)$, it follows from Equation 5 that $F(\tau_j) = 1 - e^{-\lambda_j \tau_j}$, $j = 0, 1$, and therefore

$$F(T) = \frac{1 - e^{-\lambda_0 \tau_0}}{1 - e^{-\lambda_0 \tau_0} + e^{-\lambda_1 \tau_1}} \tag{15}$$

Finally, use of Equations 9, 12, and 15 leads to the following:

$$E(H) = \frac{\left[\frac{1}{\lambda_0} e^{-\lambda_1 \tau_1} + \frac{1}{\lambda_1}\left(1 - e^{-\lambda_1 \tau_1}\right)\right]\left(1 - e^{-\lambda_0 \tau_0}\right) + \frac{1}{\mu} e^{-\lambda_1 \tau_1}}{1 - e^{-\lambda_0 \tau_0} + e^{-\lambda_1 \tau_1}} \tag{16}$$

Notice that Equation 16 behaves as expected in the limit cases. As $r \rightarrow 0$, $e^{-\lambda_j \tau_j} \rightarrow 1$, and therefore $E(H) \rightarrow 1/\mu$ (i.e., in the absence of SVs capacity is maximal). Similarly, as $r \rightarrow 1$, $E(H) \rightarrow 1/\lambda_1$ (i.e., when there are many SVs capacity tends to the minimum possible).

The case $n = 1$ is particularly simple because $\lambda_0 \tau_0 = \lambda_1 \tau_1 = rL\kappa$. In this case, Equation 16 simplifies to

$$E(H) = \frac{1}{\lambda_0} e^{-rL\kappa}\left(1 - e^{-rL\kappa}\right) + \frac{1}{\lambda_1}\left(1 - e^{-rL\kappa}\right)^2 + \frac{1}{\mu} e^{-rL\kappa} \tag{17}$$

which highlights the fact that the expected headway is a weighted average of $1/\lambda_0$, $1/\lambda_1$, and $1/\mu$.

Model M3: Trivalued Disturbances

This model supposes that given that the SV arrival takes place within a queue, its disturbance can take two values, τ_1 and τ_2, with probabilities α and $1 - \alpha$, respectively. For arrivals during free flow the disturbance will be, as in Model M2, τ_0; that is,

$$\begin{aligned} P(T_i = \tau_0) &= 1 - F(T_{i-1}) \\ P(T_i = \tau_1) &= \alpha F(T_{i-1}) \\ P(T_i = \tau_2) &= (1 - \alpha) F(T_{i-1}) \end{aligned} \tag{18}$$

The aim of this model is to recognize that the slow portion of the leading SV's trajectory (vehicle $i - 1$) may have different speeds and therefore may induce a different disturbance time to upstream SV *i*. Proceeding analogously to Equation 13 and imposing the steady-state condition $F(T_i) = F(T_{i-1}) = F(T)$ leads to the following:

$$F(T) = \frac{1 - e^{-\lambda_0 \tau_0}}{2 + e^{-\lambda_0 \tau_0} - \alpha\left(1 - e^{-\lambda_1 \tau_1}\right) - (1 - \alpha)\left(1 - e^{-\lambda_2 \tau_2}\right)} \tag{19}$$

Finally, the expected SV headway is given by

$$\begin{aligned} E(H) = &E(H \mid T = \tau_0)\left[1 - F(T)\right] \\ &+ F(T)\left[\alpha E(H \mid T = \tau_1) + (1 - \alpha) E(H \mid T = \tau_2)\right] \end{aligned} \tag{20}$$

Model M4: Continuous Disturbances

This model is the continuous limit of Model M3 and assumes that arrivals in congestion may produce disturbances in $\tau_0 < T_i \leq \tau_1$ with equal probability; that is,

$$\begin{aligned} P(T_i = \tau_0) &= 1 - F(T_{i-1}) \\ P(\tau_0 < T_i \leq \tau_1) &= \frac{F(T_{i-1})}{\tau_1 - \tau_0} \end{aligned} \tag{21}$$

Analogously to Equation 13 in the steady state,

$$F(T) = F(\tau_0)\left[1 - F(T)\right] + \frac{F(T)}{\tau_1 - \tau_0} \int_{\tau_0}^{\tau_1} F(u)\, du \tag{22}$$

Since the integral $\int_{\tau_0}^{\tau_1} F(u)\, du = (\tau_1 - \tau_0) - \frac{e^{-\lambda_0 \tau_0} - e^{-\lambda_1 \tau_1}}{rQ_D}$,

$$F(T) = \frac{1 - e^{-\lambda_0 \tau_0}}{1 - e^{-\lambda_0 \tau_0} + \frac{e^{-\lambda_0 \tau_0} - e^{-\lambda_1 \tau_1}}{rQ_D(\tau_1 - \tau_0)}} \tag{23}$$

The average headway can now be computed as

$$E(H) = E(H \mid T = \tau_0)\left[1 - F(T)\right] + \frac{F(T)}{\tau_1 - \tau_0} \int_{\tau_0}^{\tau_1} E(H \mid T = u)\, du \tag{24}$$

Unfortunately, the integral $\int_{\tau_0}^{\tau_1} E(H \mid T = u)\, d_i$ has no analytical solution since it involves the exponential integral $\int u e^{-u} du$, which can only be evaluated numerically. However, since $E(H \mid T)$ is a slow-varying function, the following approximation turns out to be surprisingly accurate:

$$\frac{1}{\tau_1 - \tau_0} \int_{\tau_0}^{\tau_1} E(H \mid T = u)\, du \approx \frac{E(H \mid T = \tau_0) + E(H \mid T = \tau_1)}{2} \tag{25}$$

In fact, a comparison of this approximation with the numerical evaluation of the original integral indicates that values of ρ coincide up to the third decimal point.

EXAMPLE: CAPACITY OF SHORT UPHILL GRADES

This section shows the performance of Models M1 through M4 for the case of short uphill grades and compares them with the simulation results of the SPH model (*8*). Simulation was chosen as a benchmark because empirical data are not available. In fact, the recommendations in the 2000 *Highway Capacity Manual* (*9*) for the capacity of upgrades are based on the microsimulation model FRESIM, which gives debatable results (*8*).

The SPH model simulates exactly the real-world process approximated in this paper (i.e., truck trajectories are realistic and their disturbances depend on their speed at x_0, as in Figure 2). This section shows that the approximate trajectories in Figure 3 together with a suitable distribution for disturbance times allow the derivation of closed-form expressions for the capacity, which match the values obtained via the simulation of the "exact" problem. The following section briefly presents the SPH model.

SPH Modeling

Let $x = \phi(t)$ be the position, x, of a moving bottleneck (truck) at time t. Its desired acceleration $a[v(t), x]$ is given by the kinetics model incorporated in TWOPAS (*10*):

$$a = \frac{\beta a_p}{\beta + 1.5\ S(a_p)(a_p - a_c)} \quad (\text{in ft/s}^2) \tag{26}$$

where $S(a_p) = -1$ if $a_p < 0$ and $S(a_p) = 1$ otherwise; the rest of the parameters are defined as follows:

$$\beta = \begin{cases} 0.4v & \text{if } v < 10, \\ 10 & \text{otherwise} \end{cases}$$

$$a_p = \frac{a_c + 15{,}368\ \dfrac{C_p}{W} v}{1 + \dfrac{14{,}080}{W} v^2}$$

$$a_c = -0.2445 - 0.0004v - \frac{0.021\ C_d v^2}{A} - \frac{222.6 C_p}{W} v - gG \tag{27}$$

$$C_p = 1 - 0.00004d$$

$$C_d = (1 - 0.00000688d)^{4.255}$$

where d is expressed in ft, v in ft/s, W in lb/hp, A in ft^2/hp, and G as a decimal; the acceleration of gravity is $g = 32.17$ ft/s^2. The other parameters are weight-to-power ratio, W; frontal area-to-power ratio, A; grade at location x, $G(x)$; and altitude, $d(x)$. The truck's current speed, $v(t)$, is constrained both by its ability to accelerate and by traffic downstream:

$$v(t) = \min[v_{\text{dwn}}(t),\ v_{\text{des}}(t)] \tag{28}$$

where $v_{\text{dwn}}(t)$ is a numerical estimate for the speed of the KW stream immediately downstream of the truck in $[t, t + \Delta t)$ and $v_{\text{des}}(t)$ is the desired truck speed, obtained with the relation

$$v_{\text{des}}(t + \Delta t) = v(t) + a[v(t),\ \phi(t)]\Delta t \tag{29}$$

The truck's position is then updated with $\phi(t + \Delta t) = \phi(t) + v(t)\Delta t$. There are several numerical solution methods to solve the KW-MB problems so generated. The method of Daganzo and Laval (*11*) was chosen for its simplicity.

Experiments

In this example an uphill segment of length $L = 0.1$ mi is considered. Each lane of the segment has a triangular FD with free-flow speed $u = 60$ mph, wave velocity $w = 15$ mph, and jam density for one lane $\kappa = 150$ vpmpl. Two truck types are considered separately: heavy (light) trucks with weight-to-power ratios of 228 (140) lb/hp and weight-to-frontal-area ratios of 682 (312) lb/ft^2. The effects of the altitude $d(x)$ have been neglected. Three experiments are considered:

Experiment 1. $G = 6.5\%$, $n = 2$ lanes, light truck;
Experiment 2. $G = 6.5\%$, $n = 2$ lanes, heavy truck; and
Experiment 3. $G = 4.5\%$, $n = 2$ lanes, heavy truck.

In all the experiments the truck proportions vary in $0 \le r \le 10\%$ in satisfying the condition of r being small. The parameters for models M1 through M4 are chosen so that no calibration is necessary. Therefore, τ_0 is chosen as the disturbance caused by a truck passing through the bottom of the uphill at free-flow speed. To compute this value one must evaluate Equation 2 with $L = 0.1$ mi and v resulting form the kinetics model (Equations 26 and 27), which gives $v = \{51.4, 48.0, 55.4\}$ for Experiments 1, 2, and 3, respectively. Similarly, τ_1 is taken as the disturbance when the truck travels the entire uphill at the crawl speed, in which case the vehicle kinetics model gives $v_C = \{31.1, 19.0, 26.2\}$. For τ_2 the mean between the maximum and minimum disturbances is chosen [i.e., $(\tau_0 + \tau_1)/2$]. Finally, in the absence of more information the parameter α is taken as one-half.

Results

Figure 5 shows the estimates for the dimensionless capacity as a function of the truck percentage for Models M1 through M4 (lines) and for the simulation (circles). For all three experiments the conclusions are similar. As anticipated earlier, Model M1 seriously underestimates capacity because of its pessimistic assumptions on disturbance times. Model M2 improves the estimation especially for $r < 3\%$, but for larger truck proportions the quality of the estimation diminishes. Model M3 exhibits a significant improvement over M2 for all the values of r with a maximum deviation from simulation results of 2 percentage points, which is good. The best fit, however, is obtained with Model M4, which replicates almost exactly the simulation benchmark. Moreover, this model requires the fewest parameters (excluding Model M1), and they are easily observable. These results highlight the importance of including more than one disturbance time in the estimation of short uphill grade capacity. In addition, the good results of Model M4 indicate that the disturbance distribution in congestion is well approximated by a uniform distribution.

DISCUSSION OF RESULTS

The disturbance distributions in congestion proposed in this paper are aggregate distributions as opposed to distributions for a particular SV. As such, they must be interpreted as the result of a large sample of the actual disturbances rather than as the result of microscopic considerations. In the microscopic approach the disturbance of a given SV will be the maximum between its desired disturbance (drawn from a desired, or feasible, disturbance distribution) and the disturbance dictated by the SV downstream, which in turn obeys a maximum rule. This complicates matters because the actual distribution will depend nonlinearly on the desired distribution and on the headway distribution, and closed-form expressions may not be found.

The fact that the uniform distribution (see Equation 21) provides an excellent estimate for the uphill example is therefore an important result because it validates the use of aggregate distributions, which allow for mathematical tractability. It also suggests that the case of several SV types may be tackled similarly, as well as any bottleneck caused by the presence of SVs. This is under investigation by the author.

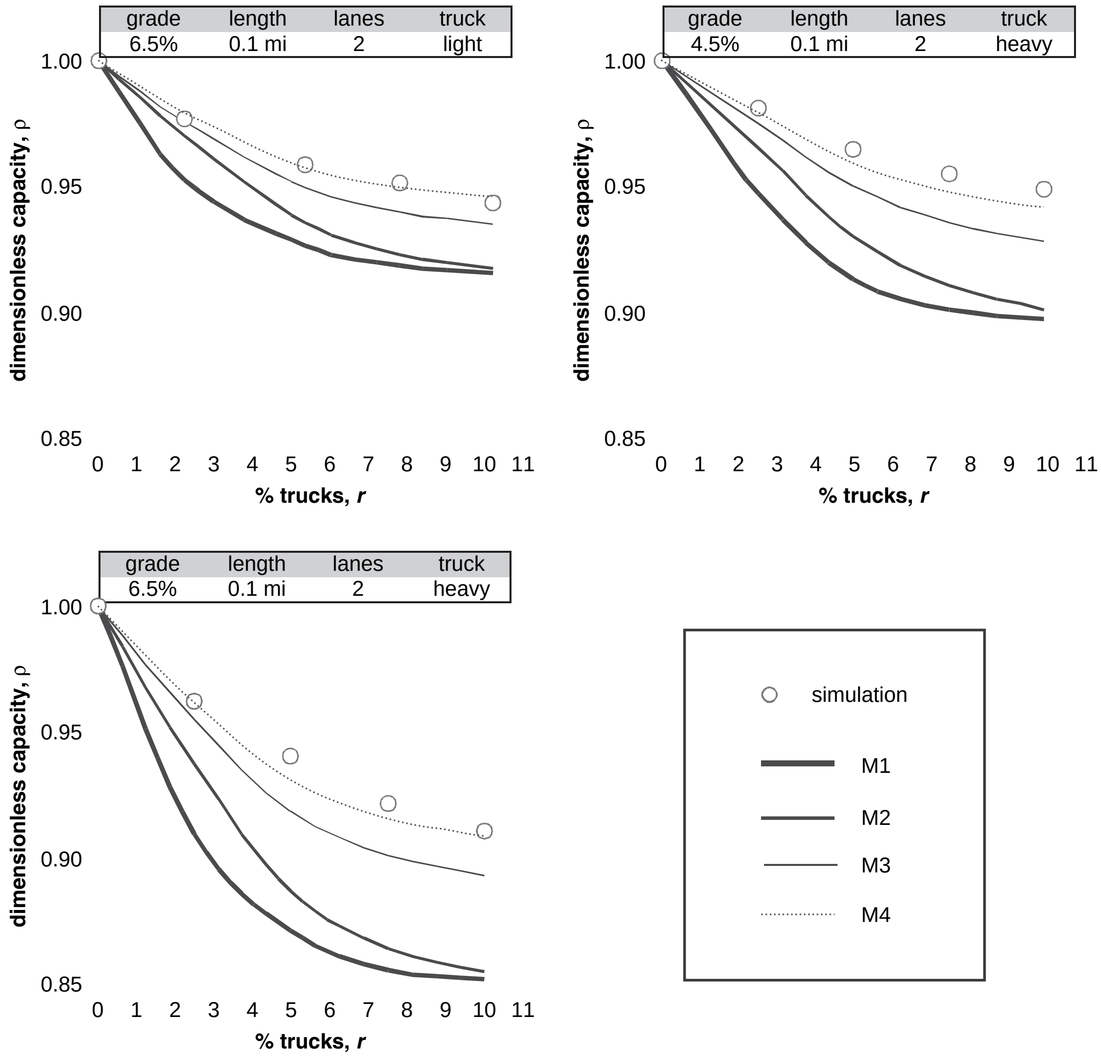

FIGURE 5 Estimates of the dimensionless capacity as a function of the truck percentage for Models M1 through M4 (lines) and for the simulation (circles) for all three experiments.

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